Computational study on two-phase MHD buoyancy driven flow in an asymmetric diverging channel

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Abstract

In this paper the problem of a two-dimensional steady viscous, incompressible two-phase flow of a particulate suspension in an asymmetric diverging channel with a heat source is considered. The differential equations governing the flow are non-dimensionalized by employing suitable transformations and resulting equations are solved numerically, using Runge-Kutta Shooting technique. The influence of Magnetic parameter, Reynolds number, Cross flow Reynolds number, Grashof number, heat source parameter, Prandtl number are exhibited graphically and velocity and temperature profiles for both fluid as well as particle phases discussed. Computational values for skin friction coefficient, Nusselt number are obtained and presented in tabular form and discussed. This study plays an important role in many engineering and biological fields such as cooling of nuclear reactors, chemical and food industries, blood flow through capillaries and arteries.

Keywords: MHD, particulate suspension, two-phase flow, diverging channel

1. Introduction


Study of electrically conducting fluid flowing through channels is useful in industrial and biological systems such as MHD accelerator technologies and hydro-magnetic energy generators. Alam and Khan (2014) used finite element method to investigate MHD effects on mixed convective flow through a diverging channel with circular obstacle. Mir, Umar, Naveed, Raheela, and Syed (2013) investigated MHD flow of a Jeffery fluid in converging and diverging channels. Hatami, Sheikholeslami, Hosseini, and Ganji (2014) studied

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Two-phase flow of particulate suspension applications abound in many areas of technology: food industries, powder technology, waste water treatment, combustion and corrosive particles in engine oil flow etc. So it is important to study fluid-particle hydromagnetic convective flows in order to understand the influence of the different phases on heat transfer processes. Recently, a remarkable number of researchers, Sivakumar, Sreenath, and Pushpavanam (2010), Hatami, Hosseinzadeh, Doma iry, and Behnamfar (2014) have investigated two-phase particulate flows with and without magnetic field and heat transfer analytically and numerically. Chamkha (1995) studied hydromagnetic two-phase flow in a channel. Mansour and Chamkha (2003) developed a continuum model to analyze heat generation effects on two-phase particulate suspension MHD flow through a channel. Usha, Senthilkumar, and Tulapurkara (2006) investigated particulate suspension flow in a travelling wavy channel. Heat generation effects on hydromagnetic flow of a particulate suspension through isothermal-isoflux channels was investigated by Chamkha and Rashidi (2010). Rawat et al. (2014) presented a numerical model for steady two dimensional two-phase hydromagnetic flows and heat transfer in a particulate-suspension through a non-Darcian porous channel. Sadia, Naheed, and Anwar (2017) studied compressible dusty gas along a vertical wavy surface. Krupalakshmi, Gireesha, Gorla, and Mamunthesh (2016) investigated numerically laminar boundary layer flow heat and mass transfer of two-phase particulate suspension past a stretching sheet with chemical reaction. Mohammad, Islam, Prilal, Ramzan, and Abumandou n (2015) investigated peristaltic transport of a particle-fluid suspension in a planar channel by taking slip effects on the wall into account. Eldesoky, Abdelsalam, Abumandou n, Kamel, and Vafai (2017) analytically studied interaction between compressibility and particulate suspension on peristaltically driven flow in a planar channel. Malikarjuna, Rashad, Hussein, and Hariprasad (2016) studied numerically the effects of transpiration, thermal radiation and thermophoresis effects on convective flow over a rotating cone in a non-Darcy porous medium. Recently Ramprasad, Subba Bhatta, and Malikarjuna (2018) considered velocity and temperature slip effects and studied numerically particulate suspension flow in a divergent channel.

With the available literature and to the best of the authors’ knowledge, no one has studied convective two-phase flow in an asymmetric divergent channel. Keeping in view the above facts, a mathematical model has been developed to study MHD convective two-phase particulate suspension flow in a divergent channel with heat source.

2. Model of the Problem

Consider steady, viscous, two-dimensional incompressible laminar two-phase flow of particulate suspension in an asymmetric divergent channel. Walls of the channel are placed at \( \theta = \pm \alpha \) as shown in Figure 1. Suction/injection velocities are assumed at different walls and these velocities are to be varied inversely proportional to the distance along the wall from origin of the channel. The continuity equation, the Navier–Stokes equations and the energy equation governing the flow in polar coordinates \((r, \theta)\) are given by Terril (1965) and Baris (2003).

For fluid phase

\[
\frac{\partial}{\partial r} \left( r u \right) + \frac{\partial}{\partial \theta} \left( r v \right) = 0, \tag{1}
\]

\[
\left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right) = \frac{1}{\rho} \left( \frac{\partial P}{\partial r} + \rho u \right) + \frac{1}{\rho} \left( \frac{\partial P}{\partial \theta} + \rho u \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right) \left( \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right) \left( \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right), \tag{2}
\]

\[
\left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} + \frac{u}{r} \right) = \frac{1}{\rho r} \left( \frac{\partial P}{\partial r} + \rho u \right) + \frac{1}{\rho r} \left( \frac{\partial P}{\partial \theta} + \rho u \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right) \left( \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right) \left( \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} \right) + \frac{\rho}{\rho} \left( \frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial \theta^2} \right) \left( \frac{\partial u}{\partial r} + \frac{v \partial u}{r} \right), \tag{3}
\]

\[
\frac{\partial}{\partial r} \left( r T \right) + \frac{\partial}{\partial \theta} \left( r v \right) = \frac{k}{\rho c_p} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right) + \frac{Q}{\rho c_p} + \frac{\rho C_p}{\tau_p} \left( T - T_0 \right), \tag{4}
\]

For particle phase

\[
\frac{\partial}{\partial r} \left( r u_p \right) + \frac{\partial}{\partial \theta} \left( r v_p \right) = 0, \tag{5}
\]
Equation (1) – (9) are reduced to
\[ f'' + 2\text{Re} f' - Rf' + L\beta (g' - f') + (4 - M^2) f' - \frac{Gr}{\text{Re}} h' = 0, \]
\[ g'' - 2\frac{Re}{R} g' - \frac{\beta^2}{R} (f' - g') - \frac{Gr}{\text{Re} R} h' = 0, \]
\[ h' - R Pr h' + Pr Q h' + Lf' g Pr (H - h) = 0, \]
\[ H' - K (h - H) = 0. \]

Associated boundary conditions are
\[ f(\pm\alpha) = 0, f(0) = 1, \]
\[ g(\pm\alpha) = 0, h(\pm\alpha) = 1, H(\pm\alpha) = 1. \]

### 3. Skin Friction Coefficient and Nusselt Number

The main aim of the physical interest of the problem is analyzing drag coefficient and rate of heat transfer over surface of the channel, which are defined by skin friction coefficient and Nusselt number
\[ C_f = \frac{\tau_w}{\rho U_o^2}, \quad \text{and Nusselt number} \]
\[ Nu = \frac{\tau_w}{\kappa T_o} \]

In non-dimensional form
\[ C_f = \frac{1}{\text{Re}} f'(\pm\alpha) \quad \text{and} \quad Nu = -h'(\pm\alpha). \]

### 4. Solution Methodology

A set of equations (10) – (13) with boundary con-ditions (14) are first rewritten in a system of first order equations by assuming \( f = f(1), f' = f(2), f'' = f(3), g = f(4), g' = f(5), h = f(6), h' = f(7), H = f(8), H' = f(9), \) i.e.

\[
\begin{bmatrix}
    f(1) \\
    f(2) \\
    f(3) \\
    f(4) \\
    f(5) \\
    f(6) \\
    f(7) \\
    f(8) \\
    f(9)
\end{bmatrix} = \begin{bmatrix}
f(2) \\
f(3) \\
f(3) \\
f(4) \\
f(5) \\
f(6) \\
f(7) \\
f(8) \\
f(9)
\end{bmatrix}
\]

\[
\frac{dY}{dx} = \begin{bmatrix}
f(1) \\
f(2) \\
f(3) \\
f(4) \\
f(5) \\
f(6) \\
f(7) \\
f(8) \\
f(9)
\end{bmatrix} = \begin{bmatrix}
f(2) \\
f(3) \\
f(3) \\
f(4) \\
f(5) \\
f(6) \\
f(7) \\
f(8) \\
f(9)
\end{bmatrix}
\]

We choose some initial conditions \( f'(-\alpha) = c_1, f''(-\alpha) = c_2, g'(-\alpha) = c_3, h'(-\alpha) = c_4, \) and \( H'(-\alpha) = c_5 \) which are not given at initial point and integrate (15) using Runge-Kutta fourth order technique (Ghadikolaei, Hosseinzadeh, & Ganji, 2018;
Gholinia, Gholinia, Hosseinzadeh, & Ganji, 2018; Mallikarjuna, Rashad, Chamkha, & Hariprasad Raju, 2016; Mallikarjuna, Rashad, Hussein, & Hariprasad Raju, 2016). The obtained results were compared at α and differences attributed wrong assumptions of the initial conditions. To overcome the problem, we applied Newton Raphson method to choose the initial conditions and integrated Equation (15) using RK4 with a step size of 0.001 with $10^4$ accuracy for the solutions. To validate the present code, the results are compared in the absence of energy equation, thermal buoyancy and heat source with existing results produced by Ramprasad, Subba Bhatta, Mallikarjuna, and Srinivasacharya (2017). The obtained results were found to be good agreement as shown in Table 1. In this section we studied the role of non-dimensional flow parameters embedded in the flow model on fluid and particle phase velocities and temperatures.

5. Results and Discussion

For numerical calculations we fixed the non-dimensional parameter values as $R=1$, $Re=0.5$, $M=1.5$, $Gr=5$, $L=1$, $Pr=0.71$, $Q=0.5$, $\alpha = \pi$, $\gamma = 0.5$, $\beta = 1$, $\beta = 0.5$. These values were maintained constant in the whole study excluding dissimilarities in the particular figures.

Figures 2-4 depict the profiles of fluid velocity with variations in different governing parameters $M$, $R$, and $Gr$. As $M$, the Lorentz force which opposes the flow, increases the velocity of the fluid phase decreases as shown in Figure 2. This is in good agreement with Chamkha and Rashidi (2010), and Mir Asadullah, Umar, Naveed, Raheela, and Syed (2013). It can be concluded that the flow can be controlled by imposing higher magnetic field on the boundaries. Figure 3 illustrates that with an increase in $R$ the velocity of the fluid decreases in the left half of the channel whereas it increases in right half of the channel enormously. Near the wall $\alpha = \pi$ the viscosity effects are very small therefore the velocity attains maximum near that wall. This is in good agreement with Roy and Nayak (1982), and Terrel (1965). Figure 4 always that an increase in $Gr$ leads to increase in gravitational force and dominates the thermal buoyancy force. It causes a decrease in the fluid velocity throughout the channel.

Figures 5 to 7 represent the variation of particle phase velocity with different variations in governing parameters $M$, $R$, and $Gr$. From Figure 5 it is evident that with a hike in $M$, an enhancement of the particle phase velocity is observed in the left part of channel and opposite behavior is observed in right part of the channel. This is in good agreement with Mansour and Chamkha (2003). From Figure 6 it is observed that with an increase in $R$ the particle phase velocity increases in the left half of the channel and the reverse behavior is observed in the right half of the channel. Figure 7 shows that with an increase in $Gr$, the thermal buoyancy effect increases. This gives rise to accelerated particle phase velocity in the entire channel. The same observation has been reported by Chamkha and Rashidi (2010).

The influence of Prandtl number $Pr$, on fluid temperature is depicted in Figure 8. It can be seen from this figure that the temperature of fluid increase rapidly with an increase in $Pr$. This indicates that momentum diffusivity dominates over the fluid temperature. If $Pr=0.6$ the fluid is oxygen and if $Pr=0.71$ it is air. If $Pr=1.3$ the fluid is gaseous ammonia. Figure 9 anticipates the behavior of heat source parameter $Q$ on fluid phase temperature. As $Q$ increases the fluid phase temperature increases and maximum temperature is attained in the mid region of the channel. Figure 10 explains the behavior of $R$ on fluid temperature. As $R$ increases the fluid temperature gradually decreases in the entire channel. From Figure 11 it is observed that with an increase in $Pr$ the temperature of the particle phase increases over the left part of channel and decreases in the right part of the channel. This means that momentum diffusivity in greater in the right part of the channel and lower in the left part of the channel. An increment in $Q$ enhances the particle temperature in the left side of the channel and opposite trend is observed in the right part of the channel as demonstrated in Figure 12. Figure 13 demonstrates that with an increase in $R$ the particle phase temperature decreases in the left half of the channel and increases in the right half of the channel.

From Table 2 it is observed that with an increase in $R$, the skin friction coefficient (Cf) decreases and the Nusselt number (Nu) is enhanced near both the walls. As $Re$ increases the skin friction coefficient decreases on left wall and increases on right wall with no change in Nusselt number over both walls. As $Re$ increases the skin friction coefficient decreases near both the walls, but Nu remains constant at both the walls. This indicates that interaction of fluid and particles does not influence the rate of heat transfer over the walls. As $Re$ skin friction coefficient increases on the left wall and decreases on right, the Nusselt number decreases at both the walls.

<table>
<thead>
<tr>
<th>R</th>
<th>e</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$f'(\alpha)$</th>
<th>$f'(\alpha)$</th>
<th>$f'(\alpha)$</th>
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</table>
Figure 2. Effect of M on fluid phase velocity

Figure 3. Effect of R on fluid phase velocity

Figure 4. Effect of Gr on fluid phase velocity

Figure 5. Effect of M on particle phase velocity

Figure 6. Effect of R on particle phase velocity

Figure 7. Effect of Gr on particle phase velocity

Figure 8. Effect of Pr on fluid phase temperature

Figure 9. Effect of Q on fluid phase temperature

Figure 10. Effect of R on fluid phase temperature

Figure 11. Effect of Pr on particle phase temperature
From Table 3 it is noted that as $\gamma$ increases the skin friction coefficient increases on the left wall and decreases on the right wall. The same behavior is observed in the Nusselt number. With an increase in L skin friction coefficient on left wall and a decrease on the right wall, the same results are observed on the Nusselt number. An increment in Q reduces the skin friction coefficient and Nusselt number at the left wall whereas the reverse behavior is observed at the right wall. As M increases skin friction coefficient decreases on both walls. Nusselt number values do not change at either wall with increasing values of M.

### 6. Conclusions

In this paper, the flow of a viscous incompressible fluid through a divergent channel in a particulate suspension with MHD and heat generation has been discussed. Numerical method has been applied to solve non-linear differential equations by non-dimensionalisation using suitable transformations. The conclusions of present study are as follows.

1) With an increase in Gr, the fluid velocity decreases whereas the particle phase velocity increases.

<table>
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<tr>
<th>R</th>
<th>Re</th>
<th>$\beta$</th>
<th>$\beta'$</th>
<th>$f'(\alpha)$</th>
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### Table 2. Skin friction and Nusselt number values for different values of R, Re, $\beta$ and $\beta'$.

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<tr>
<th>$\gamma$</th>
<th>L</th>
<th>Q</th>
<th>M</th>
<th>$f'(-\alpha)$</th>
<th>$f'(\alpha)$</th>
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### Table 3. Skin friction and Nusselt number values for different values of $\gamma$, L, Q and M.
2) An increase in M increases fluid as well as particle phase velocities.
3) An increment in M decelerates fluid velocity, but particle velocity inclines in left part of the channel and declines in the right part of the channel.
4) With an increase in Cross flow Reynolds number fluid velocity decreases near the left boundary and increases near the right boundary. Similar behavior is noted in the case of particle temperature. As R increases a decline in fluid temperature is observed.
5) With an increase in R, skin friction and Nu increases on both walls. With an increase in M the reverse behavior is observed on both walls. With an increase in $u_{j}$ skin friction and Nu increases on the left wall and decreases on the right wall.

**Nomenclature**

$r, \theta$ Polar coordinates,

$\alpha$ Angle of the channel,

$\nu$ Kinematic viscosity ($m^{2}s^{-1}$),

$\mu$ Coefficient of viscosity ($kgm^{-1}s^{-1}$)

$\rho$ Density of the fluid ($kg/m^{3}$),

$u$ Fluid phase velocity ($ms^{-1}$),

$u_{p}$ Particle phase velocity ($ms^{-1}$),

S Drag coefficient of the interaction for the force exerted by one face on the other

T Fluid phase temperature (K)

$T_{p}$ Particle phase temperature (K)

$U_{0}$ Radial velocity along center line ($LT^{-1}$).

$V_{0}$ Suction/Injection velocity at $r = r_{0}$ ($LT^{-1}$)

$\beta^{*}$ Coefficient of thermal expansion

$\sigma$ Electric conductivity of the fluid (Sm$^{-1}$)

$H_{0}$ Magnetic field intensity

$\mu_{e}$ Magnetic permeability of the fluid

$\rho_{p}$ Density of the particle ($Kgm^{-3}$)

$C_{p}$ Specific heat of the fluid ($JKg^{-1}K$)

$C_{m}$ Specific heat of the particles ($JKg^{-1}K$)

$Q_{0}$ Heat generation coefficient, ($w/m^{3}$)

K Thermal conductivity of the fluid ($w/m^{1}k^{-1}$)

Re Reynolds number ($U_{0}/\nu$)

R Cross flow Reynolds number ($V_{0}/\nu$)

$L$ Ratio of the densities of the particle and fluid phase ($\rho_{p}/\rho$)

$\beta$ Fluid particle interaction parameter for velocity ($s^{2}\nu$)

$M^{2}$ Magnetic parameter ($aH_{0}^{2}\mu_{p}^{3}\nu^{2}$)

$Gr$ Grashof number ($g\beta T_{p}^{2}\nu^{4}$)

$Pr$ Prandtl number ($\mu_{p}^{2}/\nu$)

$Q$ Heat source parameter ($Q_{0}/\rho_{p}c_{p}\nu$)

$\gamma$ Specific heat ratio ($c_{p}/c_{p0}$)

**References**


Terril, R. M. (1965). Slow laminar flow in a converging or diverging channel with suction at one wall and blowing at the other wall. Journal of Applied Mathematics and Physics, 16(2), 306-308. doi:10.1007/BF01587656


