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Original Article

Topology optimization of reinforced concrete beams by a spread-over reinforcement model with fixed grid mesh

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Abstract

For this investigation, topology optimization was used as a tool to determine the optimal reinforcement for reinforced concrete beam. The topology optimization process was based on a unit finite element cell with layers of concrete and steel. The thickness of the reinforced steel layer of this unit cell was then adjusted when the concrete layer could not carry the tensile or compressive stress. At the same time, unit cells which carried very low stress were eliminated. The process was performed iteratively to create a topology of reinforced concrete beam which satisfied design conditions.

Keywords: structural topology optimization, reinforced concrete

1. Introduction

Topology optimization has made significant changes in structural design since its introduction in 1904 by Mitchell, who demonstrated the process and created a grid-like truss structure. Since the development of the finite element method in 1952-1956 (Clough, 2001), this new numerical tool has opened up new opportunities in structural optimization; see Haftka and Grüdal (1991) for an overview. With further development of the finite element method and computer technology in the 1980s, topology optimization has emerged. A pioneering method was homogenization of structural elements by Bendsøe and Kikuchi (1988) and solid isotropic material with penalization (SIMP) by Bendsøe (1989). In 1993 Xie and Steven introduced a simple evolutionary procedure for finite element based topology optimization. For more details on the development of topology optimization, readers are referred to the book by Bendsøe (1995). In 2003, Wethyavivorn and Krich (2003) and Wethyavivorn *et al.* (2003) performed full-scale testing of optimized structural topologies. Production constraints were added to the optimization process by Surit (2004).

Concrete has been used in construction since 3000 B.C. in Egypt. Its brittle nature lowers its ability to handle tensile forces, so in many situations it requires reinforcement. In principle, concrete is placed in the compressive zone while reinforcement is placed in the tension zone. The distribution of compression and tension within the structural elements depends upon the direction and magnitude of acting forces, geometric properties, and material properties. Research using topology optimization for reinforced concrete design has however been limited. Liang used performance based optimization (PBO) to find the optimal strut-and-tie model in a reinforced concrete structure (Liang *et al.* 2000; Liang *et al.* 2002). It was however based on a single material.

According to the evolutionary structural optimization process, underutilized material will be systematically removed from the ground structure. After each round of finite element analysis, the maximum Von Mises stress will be identified and multiplied by the rejection ratio, RR, and elements with lower stresses will be removed. This process will be repeated until the end criteria are satisfied (e.g., stress, displacement). However, reinforced concrete is a composite structure with two materials; concrete and steel. The brittle nature of con-

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crete makes the Von Mises stress criteria unsuitable. Other criteria, such as Mohr-Coulomb, Drucker-Prager, or normal stress criteria are more suitable.

For this study, normal stress criteria have been adopted and extended by bi-directional topology optimization. A layer-like finite element model called Spread-over Reinforcement Model (SRM) was developed. SRM will be discussed in detail in the following sections, including case studies.

2. Topology Optimization for Reinforced Concrete Structure

Topology optimization of reinforced concrete consists of four parts: 1) Spread-over reinforcement model, 2) Fixed grid mesh finite element model, 3) Bi-directional evolutionary structure optimization, and 4) Maximum stress failure criteria for concrete.

2.1 Spread-over reinforcement model

A spread-over reinforcement model for reinforced concrete structure can be done by creating a layer-like finite element model with concrete and reinforcement layers called a "unit cell" with a fixed total thickness. Each node in the unit cell has two degrees of freedom and lies in the same plane of the unit cell. The geometric configuration of a unit cell is shown in Figure 1.

In this study, the thicknesses of all unit cells were fixed to a constant value. If t_s and t_c denote the thickness of the steel layer and concrete layer respectively, the total thickness of any unit cell is set to a constant value, $t_{e,}$ and hence

$$t_e = t_s + t_c \quad (1)$$

The stiffness of the unit cell $[k_e^i]$ is then an algebraic summation of the stiffness of the concrete layer $[k_c]$ and the stiffness of the steel reinforcement $[k_s]$, as shown in Equation (2).

$$[k_e^i] = [k_c] + [k_s] \quad (2)$$

Finite element analysis will be performed by assembly element stiffness from Equation (2) and will be solved using the equilibrium Equation (3).

$$\{F\} = [K]\{u\} \quad (3)$$

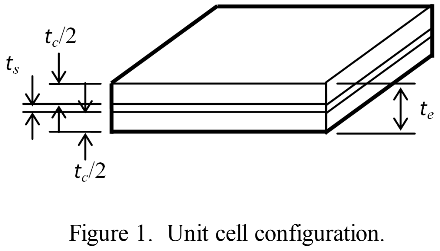


Figure 1. Unit cell configuration.

where $\{F\}$ is external forces vector, $[K]$ is global stiffness matrix, and $\{u\}$ is nodal displacements vector. Increasing the thickness of the steel layer will increase the stiffness of the unit cell.

The following assumptions are applied for the SRM method: (1) The problem was considered a two-dimensional plane stress problem, and hence the stresses σ_{33}, σ_{13}, and σ_{23} are all zero. (2) Materials, both steel and concrete, were within linear elastic limits. (3) Perfect bonding between steel and concrete layers was assumed. The effects of bound slip and shear deformation between concrete-steel contact layers were not considered in this study.

2.2 Fixed grid mesh finite element model

Fixed grid mesh uses meshing algorithms, which divide the structural domain into small elements with predefined grid size. The advantages of using fixed grid mesh are (Sigmund 2001): (1) Mesh distortion is rarely found when using fixed grid mesh in topology optimization. (2) Due to constant mesh size, stiffness matrices are similar for all elements or can be scaled if the thicknesses of some elements are not equal.

2.3 Bi-directional evolutionary structure optimization

The basic BESO procedure for a multiple–material design is an extension of the ESO procedure, which allows the addition of material to the structure at the same time as it removes inefficient material (Querin *et al.* 1998; Yang *et al.* 1999).

For this study, the optimization process was also done iteratively. Initially the thicknesses of the steel layers of all unit cells equaled zero, which is physically equivalent to starting with plain concrete. In each load step, the concrete layer in some unit cells violated normal stress criteria. The thickness of the steel layers in these unit cells was then increased until the maximum stress of the unit cell was within the prescribed limit. The minimum thickness reinforcement calculation using a bisection algorithm is described in detail in Section 2.

2.4 Maximum stress failure criteria for concrete.

With maximum stress criteria concrete would fail under two conditions: 1) Maximum tensile stress, $\sigma_{\max}^{T}$ and 2) Maximum compressive stress, $\sigma_{\max}^{C}$. Figure 2 shows the failure regions of these criteria. Stress criteria were also used as rejection criteria for some concrete elements that underwent stress lower than the prescribed values.

3. Multi-material Topology Optimization Processes

The topology optimization for reinforced concrete is an iterative one and its process is summarized below. A flow

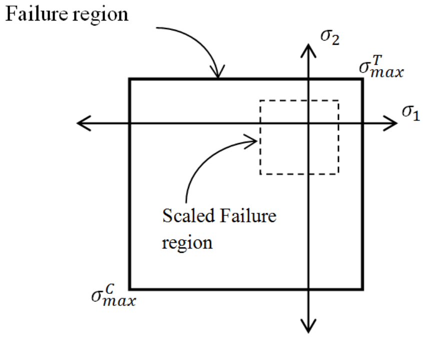


Figure 2. Maximum stress failure criteria.

chart for the process is shown in Figure 3.

1. Generate initial structure by using defined unit cell and fixed grid mesh scheme having initial thickness of reinforced steel t_s equal to zero.

2. Divide load vector $\{F\}$ into n steps, then load vector for step i defined by $\{F_0\} + i * \Delta\{F\}$, where $\{F_0\}$ is initial loading condition and $i = 1, 2, 3, \ldots, n$.

3. Perform finite element analysis for loop i to find nodal displacement vector $\{u_i\}$.

4. Compute principal stresses for all elements.

5. Detect element failure by using maximum normal stress failure criteria.

6. Find minimum reinforcement, t_s. The objective for this step is to decrease or increase the amount of reinforcing steel to the minimum volume which meets the stress requirement for all elements in the model.

6.1 Adjust t_s and then recompute nodal displacements.

6.2 Compute stress in each concrete layer and compare to prescribed failure stress. Optimal thickness for a reinforced steel layer can be found by the developed bisection algorithm.

6.3 Store the computed t_s, and repeat 6.2 until concrete layers in all unit cells pass failure criteria.

6.4 Replace t_s with new t_s from 6.3, and then update the stiffness matrix.

6.5 Repeat steps 6.1 through 6.4 until the maxi-

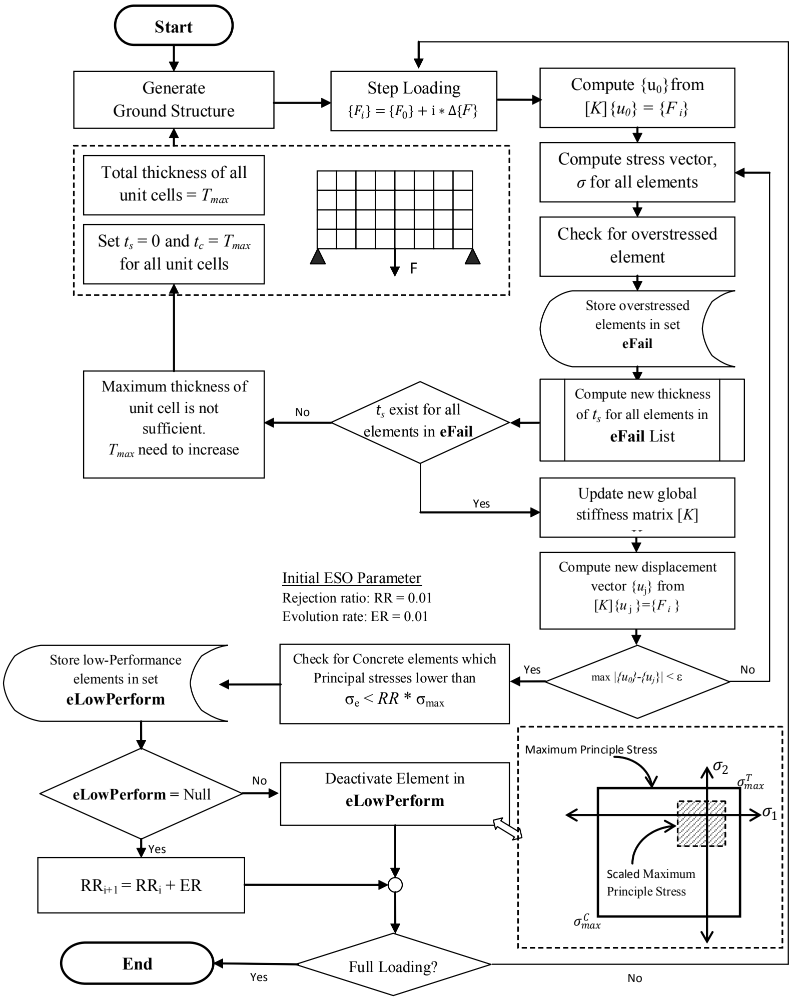


Figure 3. Optimization flow chart

mum difference t_s from the current step and the previous step for all unit cells does not change more than the defined value ε.

7. Compute stress in all unit cells and eliminate inefficient unit cells by monitoring only stresses in concrete layer.
8. Repeat steps 6 through 7 until no unit cells are removed from the model.
9. Increase the load step until full load is applied.

4. Case Study

Two case studies of simply supported beams (Figure 4) with different span to depth ratios (2 and 10) were investigated. Their material properties and model information are shown in Table 1 and 2, respectively.

5. Conclusions

The study of topology optimization of reinforced concrete beams has revealed the following:

1. By layering the material model and using the proposed process, topology optimization for multi-materials can be obtained. The optimization processes and volume of concrete in case study 1 and 2 shown in Figures 5, 7 and 9 respectively.
2. In previous studies based on single material, final reinforcement layout is a result of researcher interpretation. In this study, the proposed method leads to reinforced steel. The variable thickness of reinforcements in SRM reflects the reinforcement area required, which cannot be directly obtained from previous topology optimization processes.
3. Obtaining the optimal thickness of the steel layer in each element calculated by the bi-section method to maximize utilization of concrete is very time consuming. The computation time depends on the mesh size and the number of elements that require reinforcement.
4. For the deep beam ($L/D = 2$), the final topology of reinforced steel did not emerge, as reinforcement might not be necessary. The final topology of concrete formed an arch-like structure as shown in Figure 6. The depth was reduced from the initial design condition. However, the support condi-

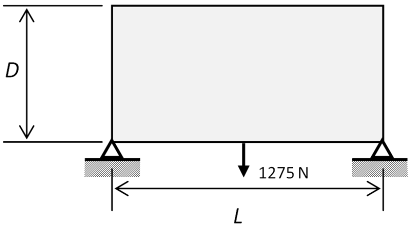


Figure 4. Simply supported beam.

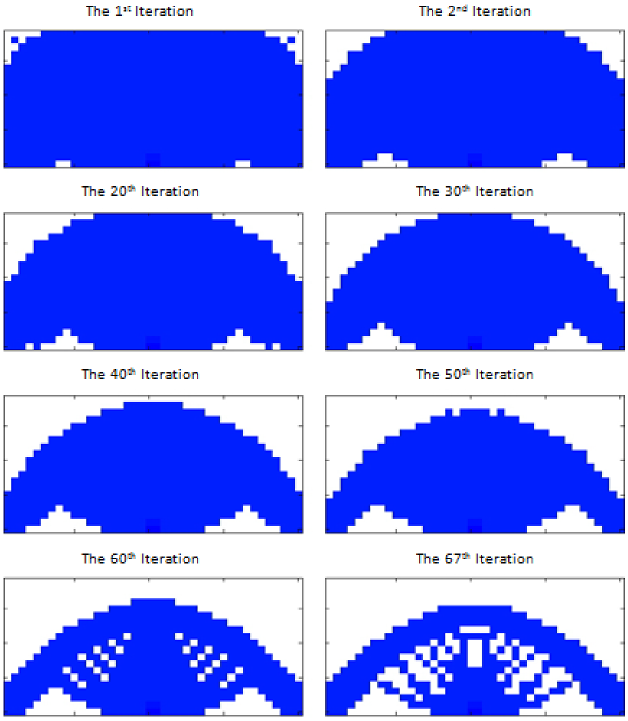


Figure 5. Optimization processes of concrete in case 1 (L/D = 2).

Table 1. Material Properties.

Material Property	Concrete	Reinforced Steel
Elastic modulus	20.594 GPa	197.11 GPa
Compressive strength	23.536 MPa	392.266 MPa
Tensile strength (from ACI $1.76\sqrt{f'_c}$)	2.677 MPa	392.266 MPa
Poisson's ratio	0.17	0.30

Table 2. Model information.

Case Study	Span, L (cm)	Depth, D (cm)	L/D ratio	t_e (cm)	Number of elements along Span	Depth
Case 1	40	20	2	1	40	20
Case 2	80	8	10	1	80	8

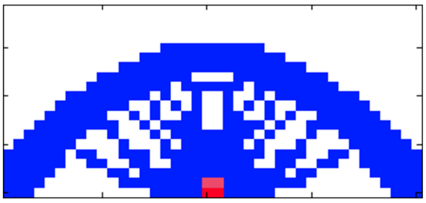


Figure 6. Final topology (the 67th Iteration) of reinforced concrete beam for case 1 (*L/D* = 2).

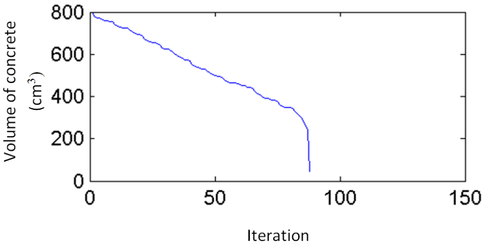
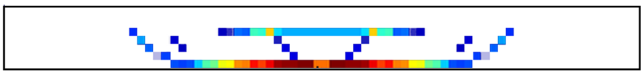


Figure 7. Iteration and volume of concrete for case 1(*L/D* = 2).

a) Final topology of concrete.



b) Final topology of reinforced steel.

c) Final Topology of reinforced concrete beam.

Figure 8. Final Topology (the 65th Iteration) of reinforced concrete beam for case 2 (*L/D* = 10).

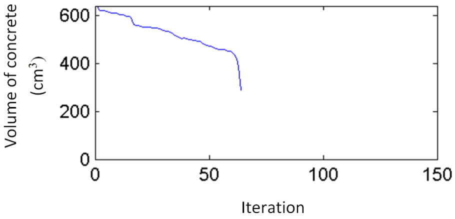


Figure 9. Iteration and volume of concrete in case 2.

tion used in this study is pinned-pinned rather than pinned-hinged and the bottom reinforcement is not present in the final topology.

5. For the normal depth beam (*L/D* = 10), the final topology of reinforced steel clearly emerged. Most of the reinforcement formed two straight lines connected together with diagonal lines as shown in Figure 8.

6. In both study cases the final topologies of reinforcement and concrete require additional processing to interpreting the actual reinforcement and geometry of concrete.

6. Future Study

As a first attempt to apply topology optimization to a two-material composite, many assumptions were involved. Future studies should include a more sophisticated numerical models computational process. The composite interaction between reinforced steel and concrete layers should also be considered. Three-dimensional models may also be developed by a similar scheme.

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Notations

C : Compliance of structure
D : Depth of beam
ER : Evolution rate
f_c : Compressive strength of concrete
$\{F\}$: Acting force vector
$\{F_0\}$: Initial acting force vector
$\{F_i\}$: Acting force vector in load step i
$[k_c]$: Stiffness matrix of concrete layer in a unit cell
$[k_e^i]$: Stiffness matrix of the i^{th} unit cell
$[k_s]$: Stiffness matrix of steel layer in a unit cell
$[K]$: Global stiffness matrix
L : Span of beam
RR : Rejection ratio
RR_i : Rejection ratio in iteration i
RR_{i+1} : Rejection ratio for the next iteration
t_c : Thickness of steel layer
t_e : Thickness of unit cell
t_s : Thickness of steel layer
T_{max} : Maximum thickness of unit cell
$\{u\}$: Displacement vector
$\{u_0\}$: Initial displacement vector
$\{u_j\}$: Displacement vector after loop j
σ_1, σ_2 : Principal stresses, tensile stress positive
σ_{ij} : Stress with $i = 1,2,3$ and $\mathrm{j} = 1,2,3$
$\sigma_{\max}^{C}$: Maximum compressive stress in concrete
$\sigma_{\max}^{T}$: Maximum tensile stress in concrete