Reed solomon inner-convolutional outer concatenated code with error-erasure decoding for narrowband and broadband power line communications

Usana Tuntoolavest1*, Narathep Sakunnithimetha1, and Kraiwee Limchaikit1

1Department of Electrical Engineering, Faculty of Engineering, Kasetsart University, Chatuchak, Bangkok, 10900 Thailand.

Received: 17 March 2016; Revised: 29 June 2016; Accepted: 7 August 2016

Abstract

A novel concatenated coding system for both narrowband and broadband power line communications (NB-PLC and BB-PLC) is proposed. The order of the inner and the outer codes switches from the G3-PLC standard and the interleaver is eliminated. The Error-and-erasure decoder (EED) is selected as the Reed-Solomon inner code is because the impulsive noises clearly mark the erasure positions with a threshold-based algorithm. The nonbinary convolutional outer code with vector symbol decoding (VSD) can correct longer burst errors. The results show that for BB-PLC, the bit error rate (BER) of RS (63,51) error-only (ED) with VSD is $1.5 \times 10^{-5}$ and of EED-VSD is $9.0 \times 10^{-9}$ at $E_b/N_0 = 26$ dB. For NB-PLC, the BER of ED-VSD is $7.9 \times 10^{-5}$ and of EED-VSD is $2.2 \times 10^{-8}$ at $E_b/N_0 = 16$ dB. Therefore, BER of EED-VSD is three to four orders of magnitude lower than ED-VSD. Moreover, NB-PLC reaches the desired BER around $10^{-7}$ at about 10 dB lower $E_b/N_0$ than BB-PLC.

Keywords: broadband, error and erasure decoding, narrowband, power line communications, vector symbol decoding, nonbinary code

1. Introduction

Power line communications (PLC) is a technology that uses power lines as the transmission media for communications. It originated from the desire for remote meter reading (Hosono, 1982). The standard “G3-PLC” is for narrowband (NB) PLC with a low frequency (under 500 kHz) and low data rate (Masood, Din, & Baig, 2013). The Broadband (BB) PLC (Tang, So, Gunawan, & Chen, 2001) allows higher data rate PLC. The main advantage of PLC is the saving on new installation costs. Its transmitters can send data through the excess bandwidth of the existing power lines that transmit electricity to buildings.

NB-PLC operates in the frequency range less than 500 kHz (Masood et al., 2013). It uses Low Voltage (LV) power lines between the local utility service and customers. Some of its applications are device-specific billing, smart energy management (Ferreira, Lampe, Newbury, & Swart, 2010) and Smart Grid (Amarsingh, Latchman, & Yang, 2014). PLC has the challenging problem of impulsive noises and non-Gaussian background noises. Originally, power lines were not intended to transmit data, but to carry electrical power. Forward error correcting code (FEC) in the G3 standard uses a concatenated code with RS (255,239) outer code and convolutional inner code as well as an interleaver to correct both random and burst errors.

BB-PLC operates in the frequency range over two MHz. It provides communication over LV or MV (Medium Voltage) power lines for in-building applications (Hasirci, Cavdar, Suljanovic, & Mujic, 2013) such as Internet, HDMI (High-Definition Multimedia Interface) Audio and games (Bert, D’Alessandro, & Tonello, 2012). IEEE 1901 (Nayagam, Rajkotia, Krishnam, & Rinchen, 2014) is a BB-PLC standard
for high-speed communication (up to 500 Mbit/s). This standard has two technologies. One uses a turbo code (Kim, 2004) and the other uses RS and convolutional, concatenated code for FEC. Both use an interleaver to convert the burst errors into random errors. Another way to correct burst errors is by turning the problem of correcting a burst of bit errors into a few erroneous, nonbinary symbols as done with RS codes. However, the typical symbol size for RS code is quite short. The G3 standard (255,239) code uses only 8 bits/symbol (Kythe & Kythe, 2012).

The proposed code is a concatenated code with a non-binary convolutional outer code and a nonbinary RS inner code. With this structure, the outer code uses larger symbols (such as 282 bits/symbols) and can correct longer bursts. The very flexible symbol size allows it to fit different channel conditions, especially in terms of burst length. The decoding system consists of an RS inner decoder with error-erasures-decoding (EED) and Vector Symbol outer Decoder (VSD). Previous work has presented this code for mobile channels (Tuntoolavest, Suktalordcheep, & Thonchai, 2013) and NB-PLC (Tuntoolavest, Sakunthinimetha, & Sompakdee, 2015; Tuntoolavest & Sompakdee, 2016). However, EED was not included due to the complexity and the small improvement in mobile channels. This paper also extends the channel to include the much more complex BB-PLC and to enable a comparison between the two PLCs.

RS code is an optimal erasure decodable code (Singleton, 1964). The accuracy of the erasure location marking is the key point in improving decoder performance (Senger, Sidorenko, Schober, Bossert, & Zyablov, 2011). This paper chooses EED as the inner decoder for PLC because it is easy and accurate to mark the erasure positions thanks to the abrupt, high-received power caused by impulsive noises. This erasure marking method is a simplified version of the threshold-based algorithm (Esquef, Biscainho, Diniz, & Freelanci, 2000). The erasure locations are marked when the received power is higher than a pre-determined threshold. The outer code is a convolutional code because it has a straightforward encoder and flexible the symbol size. The much more complicated part is the decoder. The optimal Viterbi decoding (Kovintavewat & Koonkarnkhai, 2010; Viterbi, 1967) is not practical for codes with nonbinary symbols because it requires an extremely large number of states. VSD, a suboptimal decoder, is chosen instead because it can correct any linear nonbinary codes with large enough symbols (typically 32 bits/symbol or larger).

2. Background

2.1 Reed-Solomon code

Irvine Reed and Gustav Solomon introduced Reed-Solomon (RS) codes in 1960. Such a code has the maximum possible, minimum Hamming distance \( d_{\text{min}} \) which is the Singleton bound (Singleton, 1964). A code word of an \( (n,k) \) RS code over \( \text{GF}(2^m) \) has \( n = 2^m - 1 \) total symbols, \( k \) data symbols and \( m \) bits/symbol.

Its error correction capacity \( (t) \) is

\[
t = \frac{(n-k)}{2} \text{ symbols}
\]  

EED can decode both erasures and errors at the same time. With EED, the correction capacity of RS is limited to equation (2) (Proakis, 2000). The erasure-only decoder \( (e=0) \) can correct twice the number of errors compared to the error-only decoder \( (s=0) \) as demonstrated in equation (2).

\[
e + s / 2 \leq t \text{ or } e + s / 2 \leq (n-k) / 2
\]  

where \( e \) is the number of errors and \( s \) is the number of erasures.

2.2 Vector symbol decoding

Metzner and Kapturowski proposed this decoding technique for any linear codes with nonbinary symbols in 1990 (Metzner & Kapturowski, 1990). VSD for convolutional codes (Tuntoolavest & Metzner, 2002) applied the same basic principle as VSD for block codes. Both VSD for block and convolutional codes use a parity check matrix and the syndrome computation. One main assumption of VSD is that the error values are linearly independent. This cannot be true for binary codes because the error value of an erroneous bit is always “1”. Nevertheless, when the symbol size is large such as 32 bits/symbol, the error value of an erroneous symbol can be any of the \( 2^{32} - 1 \) patterns and this assumption is easily valid. VSD is good for burst error correcting and is suitable as a concatenated outer decoder. Details of VSD for block codes are in (Vanichchanunt et al., 2009) and for convolutional codes are in (Tuntoolavest, 2009). Decoding steps are in (Tuntoolavest, 2007).

2.3 Noise characteristics and noise models of PLC

The noises in NB-PLC can be categorized into two types as shown in Figure 1a. The background noise is modeled with Additive White Gaussian Noise (AWGN) and the impulsive noise is modeled using Middleton class A (Andreadou & Tonello, 2013). These two noises cause the random and the burst errors, respectively. Therefore, the received signal \( y(t) \) can be expressed as

\[
y(t) = s(t) + n_s(t) + n_I(t)
\]  

where \( s(t) \) is the transmitted signal, \( n_s(t) \) is the AWGN and \( n_I(t) \) is the Middleton class A noise.

The more complicated noises in BB-PLC may also be categorized into two similar types as shown in Figure 1b. Specifically, the background noises consist of narrowband interferences and colored background noises while the impulsive noises consist of asynchronous and synchronous periodic impulsive noises, as well as asynchronous impulsive noises (Zimmermann & Dostert, 2000a, 2000b). The background noises can be modeled using Nakagami-m while the impulsive noise can still be modeled using Middleton class A.
noise (Andreadou & Tonello, 2013). Therefore, the received signal $y(t)$ can be expressed as

$$y(t) = s(t) + n_m(t) + n_i(t)$$

where $s(t)$ is the transmitted signal, $n_m(t)$ is the Nakagami-m noise and $n_i(t)$ is the Middleton class A noise.

### 2.4 Distributions of the noises in PLC

#### 2.4.1 Nakagami-m distribution

Meng et al. developed the background noise model for BB-PLC by measuring and analyzing the real noise values in the frequency range 1-30 MHz (Meng et al., 2004; Meng, Guan, & Chen, 2005). They discovered that the noise distribution was close to a Nakagami-m distribution. Equation 5 describes its probability density function (pdf).

$$P(r) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^m r^{2m-1} e^{-\frac{r^2}{\Omega}}$$

where $\Gamma$ is the Gamma function, $m$ is the shaping parameter and $\Omega$ is the mean background noise power.

Nakagami-m is a good model for mobile radio channels (Lu & Han, 2009) and small-scale fading channels. This is because it is more flexible than Rayleigh and Rician fading channels and sometimes matches the empirical results better (Abdi, Wills, Barger, Alouini, & Kaveh, 2000). As a background noise of BB-PLC, however, this parameter is found to be $0.5 < m < 1$ (Meng et al., 2005). For $m = 1$, the distribution is the same as for Rayleigh. With $m > 1$, it will become closer to Gaussian.

#### 2.4.2 Middleton Class A

It is difficult to model impulsive noises accurately in PLCs because they occur from different appliances such as irons, televisions and refrigerators. Different sources cause different noise characteristics. Middleton’s Class A impulsive noise model (Spaulding & Middleton, 1977) is a widely used model to generate impulsive noises. Equation 6 shows its pdf.

$$P(z) = e^{-z^2} \sum_{n=0}^{\infty} \frac{A^n}{m!} e^{-\frac{z^2}{2\sigma^2_m}}$$

where $\sigma_m^2 = \frac{A}{1+\Gamma}$ is the noise variance, $A = v/\tau_s$ is the impulse index and $v$ is the mean impulse rate and $\tau_s$ is the mean impulse duration.

Equation 6 shows that this pdf is a weighted sum of Gaussian distributions. Figure 2a illustrates an example of
impulsive noises in a time domain. Figure 2b shows that a larger impulse index \((A)\) makes the noises closer to Gaussian noises, while a smaller \(A\) makes the noises closer to Poisson process.

3. Methods

Figure 3 shows that proposed system where the outer code is a \((3,2,2)\) nonbinary, convolutional code with the generator matrix in the transform domain \(G(D)\) as follows:

\[
G(D) = \begin{bmatrix}
D^2 + D & D^2 + 1 & 1 \\
D^2 + D + 1 & D & D^2 + 1
\end{bmatrix}
\]  (7)

The inner codes are \((63, k)\) RS codes with \(k = 47, 51, 55\). Then the concatenated code word is modulated with 64-QAM (Quadrature Amplitude Modulation) and transmitted through two different channels, namely, NB-PLC and BB-PLC. The receiver demodulates the received signals and sends the output to an RS decoder. The erasure marker will also provide an appropriate erasure vector to the RS inner decoder. Figure 3 shows the input to the RS decoder for the Error-only Decoding (ED) and EED cases. Specifically, ED does not use the erasure vector, while EED uses the erasure vector to identify the positions of the RS received symbols that the decoder will erase. The decoded inner sequences become the input of VSD outer decoder. Finally, the decoded sequence from VSD is the input of the mapping circuit. This circuit converts the decoded code word to the decoded data sequence.

The simulations use MATLAB for the programming of most blocks since it has useful built-in functions. However, VSD and the mapping circuit use C++ programming because it is easier to implement onto a Field Programmable Gate Array (FPGA) board after the program is converted to C. Previous work presented some implementation of VSD on a Nanoboard 3000 (Tuntoolavest et al., 2013).

Figure 3 also shows the sequence length at each step of the block diagram for one trial in the simulations. Each concatenated code word started with 10 input data symbols at point “A” in the diagram. The code rate was 2/3 and the memory size was 2, so there were 15 data symbols plus 6 tail symbols = 21 total symbols in each outer code word at point “B”. Each symbol was then encoded with \((63, k)\) RS inner encoder with three different cases of \(k\). These RS codes were over \(GF(2^6)\). Therefore, each inner symbol consisted of 6 bits and there were 63 inner symbols in each RS code word. The number of data symbols is equal to \(k\). For \((63, 47), (63, 51)\) and \((63, 55)\) codes, there were 47, 51 and 55 data symbols in each RS code word respectively. This meant that the outer symbol size at point “A” for the three cases was \(6 \times 47 = 282\) bits/symbol, \(6 \times 51 = 306\) bits/symbol and \(6 \times 55 = 330\) bits/symbol, respectively. The total of bits of each concatenated code word at point “C” is equal to \(21 \times 63 \times 6 = 7,938\) bits.

Figure 4 illustrates the performance of inner decoding with and without erasures. For ED, only the sequence “D” is the input of the RS decoder. For EED, both sequences “D” and “E” are the input of the RS decoder. The erasure vector for an \((n, k)\) code is an \(n\)-bit vector to mark the index of the erasure symbol positions. For the example in Figure 4, it is obvious that not all errors can be marked as erasures. Therefore, the decoder needs to decode both errors and erasures at the same time. Note that each 63-symbol block is the length of a \((63, k)\) RS code word. For PLC, erasure positions in the RS received sequence can be marked with relatively high confidence because the impulsive noise, which is the main cause of burst errors, is simply detected by the abrupt, high-received power. As the accuracy of the erasure marking positions plays an important role in the improvement of the
RS decoder, it is particularly helpful for PLC. Figure 5 shows that when the inner decoder fails, it will result in an erroneous outer symbol that is the input of VSD. Thus, the inner decoding failure probability ($P_{f,\text{inner}}$) is the same as the input symbol error probability ($P_s$) of VSD.

There are two main reasons that an erasure threshold provides greater improvement in PLC than in mobile channels. First, measure the threshold value directly from the received power in PLC, while it is measured from the channel gain for a mobile channel. Second, the power of an impulsive noise is high and clear in PLC, while in a mobile channel, the noise power is not as high and the threshold depends on the signal-to-noise ratio (SNR) value (Jin & Le-Ngoc, 2010).

Simulations of this proposed coding system are to find

1. the effect of changing the shaping parameter $m$ values in BB-PLC and the impulse index $A$ values in NB-PLC to the decoding failure probabilities of different inner codes.
2. the performance of the inner code only in comparison to the complete concatenated code (inner and outer codes) in NB-PLC and BB-PLC.
3. the performance of the coding system with and without erasure decoding.

4. Results

Figure 6 shows the effect of the impulse index “$A$” in NB-PLC for various RS inner codes over GF($2^n$) with ED. It can be seen that $A = 0.1$ leads to lower inner decoding failure probabilities ($P_{f,\text{inner}}$) than when $A = 0.2$ for all codes under consideration. This is because $A = 0.1$ causes fewer impulses than $A = 0.2$ case. Therefore, the numbers of errors symbols due to burst errors are lower. In addition, code with higher redundancy leads to a better decoding performance for the same impulse index. With ED and $A = 0.1$, the RS (63,51) and RS(63,47) give $P_{f,\text{inner}}$ floor values of $8.5 \times 10^{-2}$ and $1.2 \times 10^{-2}$, respectively, at $E_b/N_0$ of 10 dB, while for RS(63,55), it is $3.8 \times 10^{-3}$, which is very high.

Figure 7 shows the effect of the shaping parameter $m$ in BB-PLC for various RS inner codes over GF($2^n$) with ED and $A = 0.1$. For BB-PLC, $m$ is in the range $0.5 < m < 1$ (Meng et al., 2005), so $m = 0.75$ is considered. The cases of $m = 0.5$
and 1 are also shown to see the bounds. For \( m = 0.75 \), \( \text{RS}(63,51) \) and \( \text{RS}(63,47) \) have \( P_{\text{f,inner}} \) floor values of \( 6.6 \times 10^{-2} \) and \( 4.8 \times 10^{-3} \), respectively, at \( E_b/N_0 \) of 26 dB. With ED, they provide a good enough \( P_s \) for VSD.

Next, the (3, 2, 2) outer convolutional code and VSD are added. In comparison, \( P_f \) of the complete concatenated code is lower than that of the inner-only coding system as illustrated in Figure 8 for NB-PLC and in Figure 9 for BB-PLC. In Figure 8, \( P_{\text{f,inner}} \) of \( \text{RS}(63, 51) \) with EED is noticeably lower than with ED. With VSD, the difference in \( P_f \) of the ED and EED cases is even more significant. For \( \text{RS}(63, 51) \) ED-VSD, the \( P_f \) floor is \( 4.1 \times 10^{-3} \) at \( E_b/N_0 \) 8 dB, while for EED-VSD, it is \( 4.0 \times 10^{-6} \) at \( E_b/N_0 \) of 10 dB. Moreover, EED-VSD reaches the same \( P_f \) at the lower \( E_b/N_0 \), than ED-VSD.

The results in Figure 9 for BB-PLC are similar to Figure 8 for NB-PLC, but it reaches the floor at much higher \( E_b/N_0 \). For \( \text{RS}(63,51) \) ED-VSD, the \( P_f \) floor is \( 9.4 \times 10^{-4} \) at \( E_b/N_0 \) 24 dB, while the \( P_f \) value for the EED-VSD case is \( 1.1 \times 10^{-6} \) at the same \( E_b/N_0 \). Therefore, \( P_f \) of EED-VSD is approximately three orders of magnitude lower than ED-VSD for BB-PLC.

In addition, Figure 10 shows that the BER of NB-PLC starts to reach the floor at 8 dB and 16 dB for ED-VSD and EED-VSD, respectively. The BER of BB-PLC starts to reach the floor at 26 dB, which is a much higher \( E_b/N_0 \) than for NB-PLC.

5. Discussion

The results show that the \((63, k)\) RS code with no outer code cannot give a \( P_f \) value of less than \( 10^{-2} \) for the given
channel condition for $k = 47, 51$ and $55$. This $P_r$ value is usually too high for practical purposes. Adding the outer coding system with VSD as the outer decoder improves the decoding performance of the RS inner coding system. When the $P_{\text{err}}$ of the RS decoder is lower either by reducing the code rate or by using EED instead of ED, the $P_r$ of the concatenated code with VSD is lower to a greater degree. EED at the inner decoder provides significantly better performance for both PLCs. The erasure marking for EED is quite straightforward and simple for PLC, so it should be included in the coding system. Moreover, NB-PLC is good at a lower SNR than BB-PLC for the selected conditions.

One interesting point is the comparison of RS(63, 51) ED and RS(63, 55) EED. The first one can correct six erroneous symbols. The second one can correct eight erasure symbols. Figures 8 and 9 show that the first one has a lower $P_r$ than the second one because not all errors can be marked as erasures correctly. Some errors are from background noise. The erasure marker may not detect and mark them by the threshold-based algorithm.

6. Conclusions

This paper proposes a novel concatenated coding system for NB-PLC and BB-PLC. For the given channel conditions, the (63, 51) RS inner–(3, 2, 2) convolutional outer code with EED inner decoding and VSD outer decoding is a good choice for both PLCs. Other RS codes and convolutional code can used the same structure, as the outer encoder and decoder are very flexible in terms of symbol size. The background noise of BB-PLC is non-Gaussian. Specifically, it follows a Nakagami-m distribution. BB-PLC requires a higher SNR to obtain a similar BER as NB-PLC.

References


