Songklanakarin J. Sci. Technol.
39 (3), 359-371, May - Jun. 2017

http://www.sjst.psu.ac.th

Original Article

Bipolar soft connected, bipolar soft disconnected and bipolar soft compact spaces

Muhammad Shabir and Ayreena Bakhtawar*

Department of Mathematics,
Quaid-i-Azam University, Islamabad, 45320 Pakistan

Received: 15 February 2016; Revised: 6 June 2016; Accepted: 14 June 2016

Abstract

Bipolar soft topological spaces are mathematical expressions to estimate interpretation of data frameworks. Bipolar soft theory considers the core features of data granules. Bipolarity is important to distinguish between positive information which is guaranteed to be possible and negative information which is forbidden or surely false. Connectedness and compactness are the most important fundamental topological properties. These properties highlight the main features of topological spaces and distinguish one topology from another. Taking this into account, we explore the bipolar soft connectedness, bipolar soft disconnectedness and bipolar soft compactness properties for bipolar soft topological spaces. Moreover, we introduce the notion of bipolar soft disjoint sets, bipolar soft separation, and bipolar soft hereditary property and study on bipolar soft connected and disconnected spaces. By giving the detailed picture of bipolar soft connected and disconnected spaces we investigate bipolar soft compact spaces and derive some results related to this concept.

Keywords: bipolar soft topology, bipolar soft disjoint set, bipolar soft open set, bipolar soft closed set, bipolar soft connected space, bipolar soft disconnected space, bipolar soft compact space

1. Introduction

We create models of reality that are improvements of aspects of the genuine world. Lamentably these scientific models are excessively convoluted and we cannot locate the exact solutions. The vulnerability or instability of information while modeling issues in social sciences, medical sciences, artificial intelligence, engineering, natural sciences, etc., makes the utilization of conventional classical method successful. Therefore, traditional set theories, which were based on the crisp and exact case may not be completely suitable for taking care of issues of ambiguity/vagueness. To surpass these instabilities, the sorts of theory have been proposed (Gau *et al.*, 1993; Pawlak, 1982; Zadeh, 1965). Yet, all these speculations have their natural troubles. The reason behind these troubles is, potentially, the insufficiency of the parameterization instrument of the theory as stated by Molodtsov (1999). Molodtsov (1999) popularized the idea of the theory of soft sets as a new, effective and stronger mathematical tool for dealing with instabilities, which is free from the above challenges. In his paper, he exhibited the crucial after effects of the new theory and effectively connected it to a few headings; for example game theory, Riemann integration, probability, smoothness of function, and so forth. Soft frameworks give an exceptionally broad system with the contribution of parameters. A lot of work on soft set theory and its application in different fields have been carried out by a number of researchers (Aktas *et al.*, 2007; Ali *et al.*, 2009, 2010, 2011; Jun, 2008; Jun *et al.*, 2008; Maji *et al.*, 2003; Shabir *et al.*, 2009). If we review the history of soft topological spaces, the foundation of which was laid by Shabir *et al.* (2011), we find many remarkable authors following them (Aygunoglu *et al.*, 2012; Cagman *et al.*, 2011; Hussain *et al.*, 2011, 2014; Khalil *et al.*, 2015; Lin, 2013; Min,

* Corresponding author.
Email address: ayreena_khan@yahoo.com

2011; Peyghan, 2013, 2014; Varol *et al.*, 2012; Zakari *et al.*, 2016 Zorlutuna *et al.*, 2012).

The concept of bipolar soft sets (a hybridization of the structure of soft set and bipolarity) with its application in decision making was introduced and discussed in detail by Shabir *et al.* (2013) and studied exhaustively by Karaaslan *et al.* (2014). A bipolar soft set is acquired by viewing not only a precisely chosen set of parameters, but also an associated set of oppositely meaning parameters called the "not set of parameters". Due to the quality of providing positive and negative aspects of information at a time, the idea of the bipolar soft set is gaining momentum among researchers. Hayat *et al.* (2015) applied the concept of bipolar soft sets to hemirings. Recently, Shabir and Bakhtawar initiated the study of bipolar soft topological spaces. They defined bipolar soft topology as a collection τ of bipolar soft sets over the universe U. Consequently, they defined basic notions of bipolar soft topological spaces, bipolar soft open and bipolar soft closed sets, bipolar soft subspace, bipolar soft closure, bipolar soft interior, bipolar soft neighborhood of a point and investigated their several properties. Further, Shabir and Bakhtawar explored and studied in detail bipolar soft separation axioms.

In the present study we initiated some new ideas in bipolar soft topological spaces such as bipolar soft connected spaces, bipolar soft disconnected spaces, bipolar soft compact spaces. Section 2, presents Preliminaries on basic concepts related to soft sets, soft topological spaces, bipolar soft sets and bipolar soft topological spaces. Section 3, is devoted towards the idea of bipolar soft disjoint sets, bipolar soft separation of a set, bipolar soft connected spaces, bipolar soft disconnected spaces, and bipolar soft hereditary property, and some examples are given for the better understanding of these ideas. Section 3, studies the concept of bipolar soft compact spaces and some results related to these concepts are exhibited. These newly defined ideas in bipolar soft topological spaces will hopefully promote the future work and studies to be held in the bipolar soft topology and can be applied effectively to cope with uncertainties.

2. Preliminaries

In this section, we recall some basic definitions and results related to bipolar soft sets, soft topological spaces and bipolar soft topological spaces. Let U be an initial universe, E be the set of parameters, $\neg E$ be the not set of parameters. Let $P(U)$ denotes the power set of U and A, B, C be non-empty subsets of E.

Definition 1. (Maji *et al.*, 2003) Let $E = \{e_1, e_2, e_3, ..., e_n\}$ be the set of parameters. The not set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, \neg e_3, ..., \neg e_n\}$ where for all i, $\neg e_i = not\ e_i$.

Definition 2. (Shabir *et al.*, 2013) A triplet (F, G, A) is called a bipolar soft set over U, where F and G are mappings, $F : A \to P(U)$ and $G : \neg A \to P(U)$ such that $F(e) \cap G(\neg e) = \varphi$ (Empty set) for all $e \in A$.

Definition 3. (Shabir *et al.*, 2013) For two bipolar soft sets (F, G, A) and (F_1, G_1, B) over a universe U, we say that (F, G, A) is a bipolar soft subset of (F_1, G_1, B), if,

1) $A \subseteq B$ and
2) $F(e) \subseteq F_1(e)$ and $G_1(\neg e) \subseteq G(\neg e)$ for all $e \in A$.

This relation is denoted by $(F, G, A) \subseteq (F_1, G_1, B)$

Definition 4. (Shabir *et al.*, 2013). Two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the universe U are said to be equal if (F, G, A) is a bipolar soft subset of (F_1, G_1, B) and (F_1, G_1, B) is a bipolar soft subset of (F, G, A).

Definition 5. (Shabir *et al.*, 2013). The complement of a bipolar soft set (F, G, A) is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where $F^c : A \to P(U)$ and $G^c : \neg A \to P(U)$ are given by $F^c(e) = G(\neg e)$ and $G^c(\neg e) = F(e)$ for all $e \in A$.

Definition 6. (Shabir *et al.*, 2013). Extended Union of two bipolar soft sets (F, G, A) and (F_1, G_1, B) over the common universe U is the bipolar soft set (H, I, C) over U where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ F_1(e) & \text{if } e \in B - A \\ F(e) \cup F_1(e) & \text{if } e \in A \cap B \end{cases}$$

$$I(\neg e) = \begin{cases} G(\neg e) & \text{if } \neg e \in (\neg A) - (\neg B) \\ G_1(\neg e) & \text{if } \neg e \in (\neg B) - (\neg A) \\ G(\neg e) \cap G_1(\neg e) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

We denote it by $(F, G, A) \tilde{\cup} (F_1, G_1, B) = (H, I, C)$

Definition 7. (Shabir *et al.*, 2013). Extended Intersection of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C) over U where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ F_1(e) & \text{if } e \in B - A \\ F(e) \cap F_1(e) & \text{if } e \in A \cap B \end{cases}$$

$$I(\neg e) = \begin{cases} G(\neg e) & \text{if } \neg e \in (\neg A) - (\neg B) \\ G_1(\neg e) & \text{if } \neg e \in (\neg B) - (\neg A) \\ G(\neg e) \cup G_1(\neg e) & \text{if } \neg e \in (\neg A) \cap (\neg B) \end{cases}$$

We denote it by $(F,G,A) \tilde{\cap} (F_1,G_1,B) = (H,I,C)$

Definition 8. (Shabir *et al.*, 2013). Restricted Union of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C) over U where $C = A \cap B$ is non-empty and for all $e \in C$, $H(e) = F(e) \cup F_1(e)$ and $I(\neg e) = G(\neg e) \cap G_1(\neg e)$.

We denote it by $(F,G,A) \cup_R (F_1,G_1,B) = (H,I,C)$.

Definition 9. (Shabir *et al.*, 2013). Restricted Intersection of two bipolar soft sets (F,G,A) and (F_1,G_1,B) over the common universe U is the bipolar soft set (H,I,C) over U where $C = A \cap B$ is non-empty and for all $e \in C$, $H(e) = F(e) \cap F_1(e)$ and $I(\neg e) = G(\neg e) \cup G_1(\neg e)$.

We denote it by $(F,G,A) \cap_R (F_1,G_1,B) = (H,I,C)$.

Remark 1. If we take $A = B = E$ in Definitions 6, 7, 8, 9 then extended union coincides with restricted union and extended intersection coincides with restricted intersection.

Proposition 1. (Shabir *et al.*, 2013). Let (F,G,A) and (F_1,G_1,B) be two bipolar soft sets over the common universe U with the universe set of parameters *E*. Then the following are true

1) $((F,G,A) \tilde{\cup} (F_1,G_1,B))^c = (F,G,A)^c \tilde{\cap} (F_1,G_1,B)^c$,
2) $((F,G,A) \tilde{\cap} (F_1,G_1,B))^c = (F,G,A)^c \tilde{\cup} (F_1,G_1,B)^c$,
3) $((F,G,A) \cup_R (F_1,G_1,B))^c = (F,G,A)^c \cap_R (F_1,G_1,B)^c$,
4) $((F,G,A) \cap_R (F_1,G_1,B))^c = (F,G,A)^c \cup_R (F_1,G_1,B)^c$.

Definition 10. (Shabir *et al.*, 2013). The Union of two bipolar soft sets (F,G,A) and (F_1,G_1,A) over the common universe U is the bipolar soft set (H,I,A) over U where for all $e \in E$, $H(e) = F(e) \cup F_1(e)$ and $I(\neg e) = G(\neg e) \cap G_1(\neg e)$.

We denote it by $(F,G,A) \tilde{\cup} (F_1,G_1,A) = (H,I,A)$.

Definition 11. (Shabir *et al.*, 2013). The Intersection of two bipolar soft sets (F,G,A) and (F_1,G_1,A) over the common universe U is the bipolar soft set (H,I,A) over U where for all $e \in A$, $H(e) = F(e) \cap F_1(e)$ and $I(\neg e) = G(\neg e) \cup G_1(\neg e)$.

We denote it by $(F,G,A) \tilde{\cap} (F_1,G_1,A) = (H,I,A)$.

Definition 12. (Shabir *et al.*, 2013). A bipolar soft set over U is said to be relative null bipolar soft set (with respect to the parameter set *A*, denoted by $(\Phi, \tilde{u}, A)$, if for all $e \in A, \Phi(e) = \varphi$ and $\tilde{u}(\neg e) = U$, for all $\neg e \in \neg A$.

The relative null bipolar soft set with respect to the universe set of parameters *E* is called the null bipolar soft set over U and is denoted by $(\Phi, \tilde{u}, E)$

A bipolar soft set (F,G,E) over *U* is said to be a non null bipolar soft set if $F(e) \neq \varphi$ for some $e \in E$.

Definition 13. (Shabir *et al.*, 2013). A bipolar soft set over *U* is said to be relative absolute bipolar soft set (with respect to the parameter set *A*), denoted by (υ, Θ, A), if for all $e \in A$, $\upsilon(e) = U$ and $\Theta(\neg e) = \varphi$, for all $\neg e \in \neg A$.

The relative absolute bipolar soft set with respect to the universe set of parameters *E* is called the absolute bipolar soft set over *U* and is denoted by (υ, Θ, E).

Obviously, a bipolar soft set (F,G,E) over U is said to be a non absolute bipolar soft set over U, if $F(e)\neq U$ for some $e\in E$.

Definition 14. (Shabir and Bakhtawar). Let (F,G,E) be a bipolar soft set over U and $u\in U$. Then $u\in(F,G,E)$ if $u\in F(e)$ for all $e\in E$.

If $u\in(F,G,E)$ then automatically $u\notin G(\neg e)$ for all $\neg e\in\neg E$.

Note that for any $u\in U$, $u\notin(F,G,E)$, if $u\notin F(e)$ for some $e\in E$.

Definition 15. (Shabir and Bakhtawar). Let Y be a non-empty subset of U. Then $(\tilde{Y},\Theta,E)$ denotes the bipolar soft set over U defined by $\tilde{Y}(e)=Y$ for all $e\in E$ and $\Theta(\neg e)=\varphi$ for all $\neg e\in\neg E$.

Definition 16. (Shabir and Bakhtawar). Let $u\in U$. Then $(F_{u,}G_{u},E)$ denotes the bipolar soft set over U, defined by $F_u(e)=\{u\}$ and $G_u(\neg e)=U\setminus\{u\}$, for each $e\in E$.

Definition 17. (Shabir and Bakhtawar). Let τ be the collection of bipolar soft sets over U with E as the set of parameters. Then τ is said to be a bipolar soft topology on U if

(1) $(\Phi,\tilde{u},E),(\upsilon,\Theta,E)$ belongs to τ

(2) the union of any number of bipolar soft sets in τ belongs to τ

(3) the intersection of any two bipolar soft sets in τ belongs to τ

Then $(U,\tau,E,\neg E)$ is called a bipolar soft topological space over U and the members of τ are said to be bipolar soft open sets in U.

Proposition 2. (Shabir and Bakhtawar). Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U. Then the collection $\tau_e=\{F(e)\,|\,(F,G,E)\in\tau\}$ for each $e\in E$, defines a topology on U.

Definition 18. (Shabir and Bakhtawar). Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U and (F,G,E) be a bipolar soft set over U. Then (F,G,E) is said to be bipolar soft closed if and only if $(F,G,E)^c$ belongs to τ.

A bipolar soft set (F,G,E) over U is said to be bipolar soft clopen if it is both a bipolar soft closed and a bipolar soft open set over X.

Definition 19. (Shabir and Bakhtawar). Let (F,G,E) be a bipolar soft set over U and Y be a non-empty subset of U. Then the bipolar sub soft set of (F,G,E) over Y denoted by $({}^{Y}F,{}^{Y}G,E)$, is defined as follows

${}^{Y}F(e)=Y\cap F(e)$ and ${}^{Y}G(\neg e)=Y\cap G(\neg e)$, for each $e\in E$.

Proposition 3. (Shabir and Bakhtawar). Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U and Y be a non-empty subset of X. Then $\tau_Y=\{({}^{Y}F,{}^{Y}G,E)\,|\,(F,G,E)\in\tau\}$ is a bipolar soft topology on Y.

Theorem 1. (Shabir and Bakhtawar). Let (U,τ_s,E) be a soft topological space over U (Shabir *et al.*, 2011). Then the collection τ consisting of bipolar soft sets (F,G,E) such that $(F,E)\in\tau$ and $G(\neg e)=F'(e)=U\setminus F(e)$ for all $\neg e\in\neg E$, defines a bipolar soft topology over U.

3. Bipolar Soft Connected and Bipolar Soft Disconnected Spaces

In this section, we discussed and explored one of the most important property of bipolar soft topological spaces called the bipolar soft connectedness and bipolar soft disconnectedness.

Definition 20. Two bipolar soft sets (F_1,G_1,E), (F_2,G_2,E) are said to be bipolar soft disjoint if $F_1(e)\cap F_2(e)=\varphi$ for all $e\in E$.

Definition 21. Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U. A bipolar soft separation of (υ,Θ,E) is a pair (F_1,G_1,E), (F_2,G_2,E) of non-null disjoint bipolar soft open sets over U such that $F_1(e)\cup F_2(e)=U$ for all $e\in E$.

Definition 22. A bipolar soft topological space $(U,\tau,E,\neg E)$ is said to be a bipolar soft disconnected space if there exists a bipolar soft separation of (υ,Θ,E).

Further, $(U,\tau,E,\neg E)$ is said to be a bipolar soft connected space if and only if it is not a bipolar soft disconnected space.

Example 1. Let $U=\{m_1,m_2,m_3,m_4\}$ be the universe set representing "markets". Let $E=\{e_1,e_2\}=\{$hand embroidery dresses, formal dresses$\}$ and $\neg E=\{\neg e_1,\neg e_2\}=\{$machine embroidery, causal dresses$\}$. Let (F_1,G_1,E), (F_2,G_2,E) represents the preferences of markets for selection of clothes by two women. Then the bipolar soft topology over U generated by (F_1,G_1,E), and (F_2,G_2,E) is given by $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E)\}$ where (F_1,G_1,E), $(F_2,G_2,E),(F_3,G_3,E)$ are bipolar soft sets over U defined as follows:

$F_1(e_1)=\{m_1\}$, $F_1(e_2)=\{m_2,m_4\}$ and $G_1(\neg e_1)=\{m_2\}$, $G_1(\neg e_2)=\{m_1\}$,

$F_2(e_1)=\{m_2,m_3,m_4\}$, $F_2(e_2)=\{m_1,m_3\}$ and $G_2(\neg e_1)=\varphi$, $G_2(\neg e_2)=\{m_2\}$,

$F_3(e_1)=\varphi$, $F_3(e_2)=\varphi$ and $G_3(\neg e)=\{m_2\}$, $G_3(\neg e)=\{m_1,m_2\}$. Then $(U,\tau,E,\neg E)$ is a bipolar soft disconnected space because (F_1,G_1,E) and (F_2,G_2,E) form a bipolar soft separation of (υ,Θ,E).

Example 2. Let $U=\{w_1,w_2,w_3\}$ be the universe set representing "wedding marques". Let $E=\{e_1,e_2,e_3\}$ = {Expensive, best food service, ideal decoration facility} be the set of parameters and $\neg E=\{\neg e_1,\neg e_2,\neg e_3\}$ = {cheap, average food service, poor decoration facility} be the not set of parameters. Let (F_1,G_1,E), (F_2,G_2,E) represents the choices made by two different families for the selection of wedding marques. Then $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),$ $(F_3,G_3,E),(F_4,G_4,E)\}$ is the bipolar soft topology over U generated by (F_1,G_1,E), (F_2,G_2,E) where

$F_1(e_1)=\{w_1,w_3\}$, $F_1(e_2)=\{w_2,w_3\}$, $F_1(e_3)=\{w_1,w_2\}$ and $G_1(\neg e_1)=\{w_2\}$,

$G_1(\neg e_2)=\{w_3,w_4\}$, $G_1(\neg e_3)=\{w_3\}$,

$F_2(e_1)=\{w_3,w_4\}$, $F_2(e_2)=\{w_1,w_2,w_3\}$, $F_2(e_3)=\{w_1,w_4\}$ and $G_2(\neg e_1)=\{w_1,w_2\}$,

$G_2(\neg e_2)=\{w_4\}$, $G_2(\neg e_3)=\varphi$

$F_3(e_1)=\{w_3\}$, $F_3(e_2)=\{w_2,w_3\}$, $F_3(e_3)=\{w_1\}$ and $G_3(\neg e_1)=\{w_1,w_2\}$,

$G_3(\neg e_2)=\{w_3,w_4\}$, $G_3(\neg e_3)=\{w_3\}$,

$F_4(e_1)=\{w_1,w_3,w_4\}$, $F_4(e_2)=\{w_1,w_2,w_3\}$, $F_4(e_3)=\{w_1,w_2,w_4\}$ and $G_4(\neg e_1)=\{w_2\}$,

$G_4(\neg e_2)=\{w_4\}$, $G_4(\neg e_3)=\phi$.

We note that the bipolar soft topological space generated by $(F_1,G_1,E),(F_2,G_2,E)$ is a bipolar soft connected space because there does not exist a bipolar soft separation of (υ,Θ,E).

Proposition 4. Let (F,G,E) be a bipolar soft set. Then

1) $(F,G,E)\cup(F^c,G^c,E)=(H,I,E)$, where $H(e)=F(e)\cup F^c(e)\subseteq U$ for each $e\in E$ and $I(\neg e)=G(\neg e)\cap G^c(\neg e)=\varphi$ for each $\neg e\in\neg E$.
2) $(F,G,E)\cap(F^c,G^c,E)=(H,I,E)$, where $H(e)=F(e)\cap F^c(e)=\varphi$ for each $e\in E$ and $I(\neg e)=G(\neg e)\cup G^c(\neg e)\subseteq U$ for each $\neg e\in\neg E$.
   Further $(F,G,E),(F^c,G^c,E)$ will always satisfy $F(e)\cup F^c(e)=G(\neg e)\cup G^c(\neg e)$ for all $e\in E$.
3) $(F,G,E)\tilde{\cap}(\upsilon,\Theta,E)=(F,G,E)$ and $(F,G,E)\tilde{\cap}(\upsilon,\Theta,E)=(\upsilon,\Theta,E)$.

Proof Straightforward.

Theorem 2. A bipolar soft topological space $(U,\tau,E,\neg E)$ is bipolar soft disconnected space if and only if there exist two bipolar soft closed sets $(F_1,G_1,E),(F_2,G_2,E)$ with $G_1(\neg e)\neq\varphi$, $G_2(\neg e)\neq\varphi$ for some $e\in E$, such that $G_1(\neg e)\cup G_2(\neg e)=U$ for all $\neg e\in\neg E$ and $G_1(\neg e)\cap G_2(\neg e)=\varphi$ for all $\neg e\in\neg E$.

Proof First, suppose $(U,\tau,E,\neg E)$ is a bipolar soft disconnected space. Then there exist a bipolar soft separation of (υ,Θ,E). Let (F,G,E) and (H,I,E) forms a bipolar soft separation of (υ,Θ,E). Then

$$F(e)\cup H(e)=U \quad \text{for all } e\in E \tag{1}$$

$$F(e)\neq\varphi \quad \text{for some } e\in E \tag{2}$$

$$H(e)\neq\varphi \quad \text{for some } e\in E \tag{3}$$

$$F(e)\cap H(e)=\varphi \quad \text{for all } e\in E \tag{4}$$

As $F(e)=G^c(\neg e)$ and $H(e)=I^c(\neg e)$, therefore from equation (1) we have $G^c(\neg e)\cup I^c(\neg e)=U$. From equation (1), (2) and (3) we have $G^c(\neg e)\cap I^c(\neg e)=\varphi$ for all $\neg e\in\neg E$, where $G(\neg e)\neq\varphi$ $I(\neg e)\neq\varphi$ for some $\neg e\in\neg E$. Further $(F,G,E)^c$ and $(H,I,E)^c$ are bipolar soft closed sets, since (F,G,E) and $(H,I,E)\in\tau$.

Conversely, suppose that there exist two bipolar soft closed sets (F_1,G_1,E), (F_2,G_2,E) with $G_1(\neg e)\neq\varphi$, $G_2(\neg e)\neq\varphi$ for some $e\in E$, such that $G_1(\neg e)\cup G_2(\neg e)=U$ for all $\neg e\in\neg E$ and $G_1(\neg e)\cap G_2(\neg e)=\varphi$ for all $\neg e\in\neg E$. Then $(F_1,G_1,E)^c$ and $(F_2,G_2,E)^c$ are bipolar soft open sets with $F_1^c(e)=G_1(\neg e)\neq\varphi$ $F_2^c(e)=G_2(\neg e)\neq\varphi$ for some $e\in E$ such that $F_1^c(e)\cup F_2^c(e)=G_1(\neg e)\cup G_2(\neg e)=U$ for all $e\in E$, and $F_1^c(e)\cap F_2^c(e)=\varphi$ for all $e\in E$. Therefore $(F_1,G_1,E)^c$ and $(F_2,G_2,E)^c$ form a bipolar soft separation of (υ,Θ,E). Thus $(U,\tau,E,\neg E)$ is a bipolar soft disconnected space.

Remark 2. The union of two bipolar soft connected spaces over the same universe need not to be a bipolar soft connected space.

Example 3. Let $U=\{h_1,h_2\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$, $\tau_1=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E)\}$ and $\tau_1=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_2,G_2,E)\}$ where $F_1(e_1)=U$, $F_1(e_2)=\varphi$ and $G_1(\neg e_1)=\varphi$, $G_1(\neg e_2)=U$, $F_2(e_1)=\varphi$, $F_2(e_2)=U$ and $G_2(\neg e_1)=U$, $G_2(\neg e_2)=\varphi$.

Then $(U,\tau_1,E,\neg E)$, $(U,\tau_2,E,\neg E)$ are bipolar soft connected spaces over U. But we note that is not a bipolar soft connected space because $(F_1,G_1,E),(F_2,G_2,E)$ form a bipolar soft separation of $(\Phi,\tilde{u},E)$ in $\tau_1\cup\tau_2$.

Proposition 5. The intersection of two bipolar soft connected spaces over a same universe is a bipolar soft connected space.

Proof. Let $(U,\tau_1,E,\neg E)$ and $(U,\tau_2,E,\neg E)$ be two bipolar soft connected spaces. Suppose to the contrary that $(U,\tau_1\cap\tau_2,E,\neg E)$ is not a bipolar soft connected space. Then there exist bipolar soft sets $(F_1,G_1,E),(F_2,G_2,E)$ belonging to $\tau_1\cup\tau_2$, which forms a bipolar soft separation of $(\Phi,\tilde{u},E)$ in $(U,\tau_1\cap\tau_2,E,\neg E)$. Since $(F_1,G_1,E),(F_2,G_2,E)\in\tau_1\cap\tau_2$ then $(F_1,G_1,E),(F_2,G_2,E)\in\tau_1$ and $(F_1,G_1,E),(F_2,G_2,E)\in\tau_1$. This implies that $(F_1,G_1,E),(F_2,G_2,E)$ form a bipolar soft separation of $(\Phi,\tilde{u},E)$ in $(U,\tau_1,E,\neg E)$ and $(F_1,G_1,E),(F_2,G_2,E)$ form a bipolar soft separation of $(\Phi,\tilde{u},E)$ in $(U,\tau_2,E,\neg E)$ which is a contradiction to given hypothesis. Thus $(U,\tau_1\cap\tau_2,E,\neg E)$ is a bipolar soft connected space.

Remark 3. The intersection of two bipolar soft disconnected spaces over the same universe need not to be a bipolar soft disconnected space.

Example 4. Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$, $\tau_1=(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E)\}$ and $\tau_2=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(H_1,I_1,E),(H_2,I_2,E),(H_3,I_3,E)\}$ where

$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_1,h_2\}$ and $G_1(\neg e_1)=\{h_2\}$, $G_1(\neg e_2)=\{h_3\}$

$F_2(e_1)=\{h_2,h_3\}$, $F_2(e_2)=\{h_3\}$ and $G_2(\neg e_1)=\varphi$, $G_2(\neg e_2)=\{h_1\}$

$F_3(e_1)=\varphi$, $F_3(e_2)=\varphi$ and $G_3(\neg e_1)=\{h_2\}$, $G_3(\neg e_2)=\{h_1,h_3\}$

$H_1(e_1)=\{h_1,h_3\}$, $H_1(e_2)=\{h_1,h_3\}$ and $I_1(\neg e_1)=\{h_2\}$, $I_1(\neg e_2)=\{h_2\}$

$H_2(e_1)=\{h_2\}$, $H_2(e_2)=\{h_2\}$ and $I_2(\neg e_1)=\{h_1\}$, $I_2(\neg e_2)=\{h_1\}$

$H_3(e_1)=\varphi$, $H_3(e_2)=\varphi$ and $I_3(\neg e_1)=\{h_1,h_2\}$, $I_3(\neg e_2)=\{h_1,h_2\}$.

Then $(U,\tau_1,E,\neg E)$, $(U,\tau_2,E,\neg E)$ are bipolar soft disconnected spaces over U.

Now, $\tau_1\cap\tau_2=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E)\}$. We note that $(U,\tau_1\cap\tau_2,E,\neg E)$ is not a bipolar soft disconnected space because there do no exist any two non null disjoint bipolar soft open sets (F,G,E) and (H,I,E) belonging to $\tau_1\cap\tau_2$ such that $F(e)\cup H(e)=U$ for all $e\in E$.

Proposition 6. The union of two bipolar soft disconnected spaces over the same universe is a bipolar soft disconnected space.

Proof Straightforward.

Theorem 3. Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U and let the bipolar soft sets (F_1,G_1,E) and (F_2,G_2,E) form a bipolar soft separation of (υ,Θ,E). If $(Y,\tau_Y,E,\neg E)$ is a bipolar soft connected subspace of $(U,\tau,E,\neg E)$ then $Y\subseteq F_1(e)$ for all $e\in E$ or $Y\subseteq F_2(e)$ for all $e\in E$.

Proof Since (F_1,G_1,E) and (F_2,G_2,E) form a bipolar soft separation of (υ,Θ,E), we have

$$U\cap(F_1(e)\cup F_2(e))=U \quad \text{for each} \quad e\in E \tag{1}$$

$F_1(e) \neq \varphi$ for some $e \in E$ (2)

$F_2(e) \neq \varphi$ for some $e \in E$ (3)

$F_1(e) \cap F_2(e) = ?$ for all $e \in E$ (4)

As $Y \subseteq U$, we have $\left({}^Y F_1, {}^Y G_1, E\right)$ and $\left({}^Y F_2, {}^Y G_2, E\right)$ are open in $\left(Y, \tau_Y, E, \neg E\right)$.

From equation (1)

$Y \cap (F_1(e) \cup F_2(e)) = Y$ for each $e \in E$.

This implies

$(Y \cap F_1(e)) \cup (Y \cap F_2(e)) = Y$ for all $e \in E$. (5)

Also from equation (4)

$(Y \cap F_1(e)) \cap (Y \cap F_2(e)) = \varphi$ for all $e \in E$.

As $\left(Y, \tau_Y, E, \neg E\right)$ is connected, so either $Y \cap F_1(e) = \varphi$ for all $e \in E$ or $Y \cap F_2(e) = \varphi$ for all e.

If $Y \cap F_1(e) = \varphi$ for all $e \in E$ then from equation (5) $Y \cap F_2(e) = Y$ for all $e \in E$ and this implies $Y \subseteq F_2(e)$ for all $e \in E$.

If $Y \cap F_2(e) = \varphi$ for all $e \in E$ then from equation (5) $Y \cap F_1(e) = Y$ for all $e \in E$ and this implies $Y \subseteq F_1(e)$ for all $e \in E$.

Remark 4. The converse of Theorem 3 does not hold in general.

Example 5. Let $U = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$, $\neg E = \{\neg e_1, \neg e_2\}$, and $\tau = \{(\Phi, \tilde{u}, E), (\upsilon, \Theta, E), (F_1, G_1, E), (F_2, G_2, E), (F_3, G_3, E), (F_4, G_4, E), (F_5, G_5, E), (F_6, G_6, E)\}$ where

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$ and $G_1(\neg e_1) = \{h_2, h_3, h_4\}$, $G_1(\neg e_2) = \{h_2, h_3, h_4\}$,

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$ and $G_2(\neg e_1) = \{h_1, h_3, h_4\}$, $G_2(\neg e_2) = \{h_1, h_3, h_4\}$,

$F_3(e_1) = \{h_3, h_4\}$, $F_3(e_2) = \{h_3, h_4\}$ and $G_3(\neg e_1) = \{h_1, h_2\}$, $G_3(\neg e_2) = \{h_1, h_2\}$,

$F_4(e_1) = \{h_1, h_2\}$, $F_4(e_2) = \{h_1, h_2\}$ and $G_4(\neg e_1) = \{h_3, h_4\}$, $G_4(\neg e_2) = \{h_3, h_4\}$,

$F_5(e_1) = \{h_1, h_3, h_4\}$, $F_5(e_2) = \{h_1, h_3, h_4\}$ and $G_5(\neg e_1) = \{h_2\}$, $G_5(\neg e_2) = \{h_2\}$,

$F_6(e_1) = \{h_2, h_3, h_4\}$, $F_6(e_2) = \{h_2, h_3, h_4\}$ and $G_6(\neg e_1) = \{h_1\}$, $G_6(\neg e_2) = \{h_1\}$.

Then $\left(U, \tau, E, \neg E\right)$ is a bipolar soft topological space over U. Also, note that $\left(F_3, G_3, E\right)$, $\left(F_4, G_4, E\right)$ form a bipolar soft separation of $\left(\aleph, \Theta, E\right)$

Now let $Y = \{h_1, h_2\}$ then $\{h_1, h_2\}$ then $\tau_Y = \{\left(\Phi, \tilde{Y}, E\right), \left(\tilde{Y}, \Theta, E\right), \left({}^Y F_1, {}^Y G_1, E\right), \left({}^Y F_2, {}^Y G_2, E\right), \left({}^Y F_3, {}^Y G_3, E\right), \left({}^Y F_4, {}^Y G_4, E\right), \left({}^Y F_5, {}^Y G_5, E\right), \left({}^Y F_6, {}^Y G_6, E\right)\}$ is a bipolar soft topology over Y, where

${}^Y F_1(e_1) = \{h_1\}$, ${}^Y F_1(e_2) = \{h_1\}$ and ${}^Y G_1(\neg e_1) = \{h_2\}$, ${}^Y G_1(\neg e_2) = \{h_2\}$,

${}^Y F_2(e_1) = \{h_2\}$, ${}^Y F_2(e_2) = \{h_2\}$ and ${}^Y G_2(\neg e_1) = \{h_1\}$, ${}^Y G_2(\neg e_2) = \{h_1\}$,

${}^Y F_3(e_1) = \varphi$, ${}^Y F_3(e_2) = \varphi$ and ${}^Y G_3(\neg e_1) = \{h_1, h_2\}$, ${}^Y G_3(\neg e_2) = \{h_1, h_2\}$,

${}^Y F_4(e_1) = \{h_1, h_2\}$, ${}^Y F_4(e_2) = \{h_1, h_2\}$ and ${}^Y G_4(\neg e_1) = \varphi$, ${}^Y G_4(\neg e_2) = \varphi$,

${}^Y F_5(e_1) = \{h_1\}$, ${}^Y F_5(e_2) = \{h_1\}$ and ${}^Y G_5(\neg e_1) = \{h_2\}$, ${}^Y G_5(\neg e_2) = \{h_2\}$,

${}^Y F_6(e_1) = \{h_2\}$, ${}^Y F_6(e_2) = \{h_2\}$ and ${}^Y G_6(\neg e_1) = \{h_1\}$, ${}^Y G_6(\neg e_2) = \{h_1\}$.

One can easily note that $Y \subseteq F_4(e)$ for all $e \in E$. But $\left(Y, \tau_Y, E, \neg E\right)$ is not a bipolar soft connected space because $\left({}^Y F_1, {}^Y G_1, E\right)$ and $\left({}^Y F_2, {}^Y G_2, E\right)$ form a bipolar soft separation of $(\tilde{Y}, \Theta, E)$

Remark 5. If there exist a non null, non-absolute bipolar soft clopen set over U, then $\left(U, \tau, E, \neg E\right)$ need not be a bipolar soft disconnected space.

Proposition 7. Let $\left(U, \tau, E, \neg E\right)$ be a bipolar soft topological space over U. If there exist a non-null, non-absolute bipolar soft clopen set $\left(F, G, E\right)$ over U such that $F(e) \cup F^c(e) = U$ for each $e \in E$ then $\left(U, \tau, E, \neg E\right)$ is a bipolar soft disconnected space.

Proof. Since (F,G,E) is a bipolar soft clopen set over U then $(F,G,E)^c$ is a bipolar soft clopen set over U. Now, by given hypothesis and by proposition 4, we have $F(e)\cup F^c(e)=U$ for all $e\in E$ and $G(\neg e)\cap G^c(\neg e)=\varphi$ for each $\neg e\in\neg E$ and

$F(e)\cap F^c(e)=\varphi$ for each $e\in E$ and $G(e)\cup G^c(e)=U$ for each $e\in E$.

Therefore, (F,G,E) and $(F,G,E)^c$ form a bipolar soft separation of (υ,Θ,E).

Thus $(U,\tau,E,\neg E)$ is a bipolar soft disconnected space.

Example 6. Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$ and $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E),(F_4,G_4,E)\}$ where

$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_1,h_3\}$ and $G_1(\neg e_1)=\{h_2\}$, $G_1(\neg e_2)=\{h_2\}$,

$F_2(e_1)=\{h_2\}$, $F_2(e_2)=\{h_2\}$ and $G_2(\neg e_1)=\{h_1\}$, $G_2(\neg e_2)=\{h_1,h_3\}$,

$F_3(e_1)=\{h_1,h_2\}$, $F_3(e_2)=U$ and $G_3(\neg e_1)=\varphi$, $G_3(\neg e_2)=\varphi$,

$F_4(e_1)=\varphi$, $F_4(e_2)=\varphi$ and $G_4(\neg e_1)=\{h_1,h_2\}$, $G_4(\neg e_2)=U$.

Then $(U,\tau,E,\neg E)$ is a bipolar soft topological space over U. Note that (F_1,G_1,E) is a non-null, non-absolute bipolar soft clopen set over U but $(U,\tau,E,\neg E)$ is not a bipolar soft disconnected space since there do not exist a bipolar soft separation of (υ,Θ,E).

Theorem 4. Let (U,τ_s,E) be a soft topological space over U and $(U,\tau_B,E,\neg E)$ be a bipolar soft topological space over U constructed from (U,τ_s,E) as in Theorem db. If (U,τ_s,E) is a soft disconnected space (Hussain, 2014) over U then $(U,\tau_B,E,\neg E)$ is a bipolar soft disconnected space over U.

Proof. Since (U,τ_s,E) is a soft disconnected space, therefore there exist non-null soft open sets (say) (F,E), (H,E) over U such that

$\tilde{U}_E=(F,E)\cup(H,E)$ and $(F,E)\cap(H,E)=\Phi_E$.

Further, (F,G,E), (H,I,E) are non-null bipolar bipolar soft open sets (because $F(e)\neq\varphi$, $H(e)\neq\varphi$) where for all $\neg e\in\neg E$, $G(\neg e)=U\setminus F(e)$ and $I(\neg e)=U\setminus H(e)$ since (F,E), (H,E) belongs to τ_s.

Now, $F(e)\cup H(e)=U$ for all $e\in E$ and $G(\neg e)\cap I(\neg e)=(U\setminus F(e))\cap(U\setminus H(e))=\varphi$ for each $e\in E$. Also, $F(e)\cap H(e)=\varphi$ for each $e\in E$ and $G(\neg e)\cup I(\neg e)=(U\setminus F(e))\cap(U\setminus H(e))=U$ for each $e\in E$. This implies that (F,G,E), (H,I,E) belonging to τ_B, forms a bipolar soft separation of (υ,Θ,E). Thus $(U,\tau_B,E,\neg E)$ is a bipolar soft disconnected space.

Proposition 8. Let $(Y,\tau',E,\neg E)$ and $(Z,\tau'',E,\neg E)$ be two bipolar soft subspaces of $(U,\tau,E,\neg E)$ and let $Y\subseteq Z$. Then $(Y,\tau',E,\neg E)$ is a bipolar soft subspace of $(Z,\tau'',E,\neg E)$.

Proof. As $Y\subseteq Z$ so $Y=Y\cap Z$. Moreover each bipolar soft open set $({}^{Y}F,{}^{Y}G,E)$ of $(Y,\tau',E,\neg E)$ is of the form ${}^{Y}F(e)=Y\cap F(e)$ and ${}^{Y}G(\neg e)=Y\cap G(\neg e)$ for all $e\in E$ where (F,G,E) is a bipolar soft open set of $(U,\tau,E,\neg E)$. Now for each $e\in E$,

$Y\cap F(e)=(Y\cap Z)\cap F(e)$ and $Y\cap G(\neg e)=(Y\cap Z)\cap G(\neg e)$.

$\Rightarrow Y\cap F(e)=Y\cap(Z\cap F(e))$ and $Y\cap G(\neg e)=Y\cap(Z\cap G(\neg e))$.

$\Rightarrow Y\cap F(e)=Y\cap {}^{Z}F(e)$ and $Y\cap G(\neg e)=Y\cap {}^{Z}G(\neg e)$ where $({}^{Z}F,{}^{Z}G,E)$ is a bipolar soft open set in $(Z,\tau'',E,\neg E)$.

Theorem 5. Let $\{(Y_\alpha, \tau_{Y_\alpha}, E, \neg E)\}_{\alpha\in J}$ be the collection of bipolar soft connected subspaces of a bipolar soft topological space $(U,\tau,E,\neg E)$. If $\cap_{\alpha\in J} Y_\alpha \neq \varphi$, then $(\cup_{\alpha\in J} Y_\alpha, \tau_{\cup_{\alpha\in J} Y_\alpha}, E, \neg E)$ is a bipolar soft connected subspace of $(U,\tau,E,\neg E)$.

Proof. Let $\{(Y_\alpha, \tau_{Y_\alpha}, E, \neg E)\}_{\alpha\in J}$ be a collection of bipolar soft connected subspaces of $(U,\tau,E,\neg E)$, such that $\cap_{\alpha\in J} Y_\alpha \neq \varphi$. Suppose that $Y = \cup_{\alpha\in J} Y_\alpha$ and $(Y,\tau_Y,E,\neg E)$ be a disconnected subspace of $(U,\tau,E,\neg E)$. Let $({}^Y F_1, {}^Y G_1, E)$, $({}^Y F_2, {}^Y G_2, E)$ be a bipolar soft separation of (Y,Θ,E). Then

$${}^Y F_1(e) \cup {}^Y F_2(e) = Y \cap (F_1(e)\cup F_2(e)) = Y \quad \text{for all } e\in E. \tag{1}$$

$$Y\cap F_1(e) \neq \varphi \quad \text{for some } e\in E. \tag{2}$$

$$Y\cap F_2(e) \neq \varphi \quad \text{for some } e\in E. \tag{3}$$

$$(Y\cap F_1(e))\cap(Y\cap F_2(e)) = Y\cap(F_1(e)\cap F_2(e)) = \varphi \quad \text{for all } e\in E. \tag{4}$$

Consider a fixed Y_α. Then from equation (1)

$$Y_\alpha \cap (F_1(e)\cup F_2(e)) = Y_\alpha \text{ for all } e\in E.$$

From equation (4)

$$Y_\alpha \cap (F_1(e)\cap F_2(e)) = \varphi \text{ for all } e\in E.$$

Since $(Y_\alpha, \tau_{Y\alpha}, E, \neg E)$ is a bipolar soft connected subspace of $(U,\tau,E,\neg E)$ so either $Y_\alpha \cap F_1(e) = \varphi$ for all $e\in E$ or $Y_\alpha \cap F_2(e) = \varphi$ for all $e\in E$.

Now there are three cases:

(*i*) $Y_\alpha \cap F_1(e) = \varphi$ for all $e\in E$ and for all $\alpha\in J$

(*ii*) $Y_\alpha \cap F_2(e) = \varphi$ for all $e\in E$ and for all $\alpha\in J$

(*iii*) for some $\alpha\in J$, $Y_\alpha \cap F_1(e) = \varphi$ and for other some $\alpha\in J$, $Y_\alpha \cap F_2(e) = \varphi$.

Case: (*i*)

If $Y_\alpha \cap F_1(e) = \varphi$ for all $e\in E$ and for all $\alpha\in J$, then $(\cup_{\alpha\in J} Y_\alpha)\cap F_1(e) = \varphi$, that is $Y\cap F_1(e) = \varphi$ for all $e\in E$. This contradicts equation (2).

Case: (*ii*)

If $Y_\alpha \cap F_2(e) = \varphi$ for all $e\in E$ and for all $\alpha\in J$, then $(\cup_{\alpha\in J} Y_\alpha)\cap F_2(e) = \varphi$, that is $Y\cap F_2(e) = \varphi$ for all $e\in E$. This contradicts equation (3).

Case: (*iii*)

As $\cap_{\alpha\in J} Y_\alpha \neq \varphi$, so there exist some $x\in Y_\alpha$ for all $\alpha\in J$. Now by equation (1) $x\in F_1(e)\cap F_2(e)$ for all $e\in E$, this implies $x\in F_1(e)$ or $x\in F_2(e)$.

If $x\in F_1(e)$ then $Y_\alpha\cap F_1(e)\neq\varphi$ and if $x\in F_2(e)$ then $Y_\alpha\cap F_2(e)\neq\varphi$. So the case (*iii*) is not possible.

Hence our suppoition is wrong and $(Y,\tau_Y,E,\neg E)$ is a bipolar soft connected subspace of $(U,\tau,E,\neg E)$.

Definition 23. A property P of a bipolar soft topological space $(U,\tau,E,\neg E)$ is said to be a bipolar soft hereditary iff every bipolar soft subspace $(Y,\tau_Y,E,\neg E)$ of $(U,\tau,E,\neg E)$ also possesses the property P.

Remark 6. The bipolar soft connectedness (respect. bipolar soft disconnected) is not a bipolar soft hereditary property.

Example 7. Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$ and $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E)\}$ where

$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_1\}$ and $G_1(\neg e_1)=\{h_2,h_3\}$, $G_1(\neg e_2)=\{h_2,h_3\}$

$F_2(e_1)=\{h_2\}$, $F_2(e_2)=\{h_2\}$ and $G_2(\neg e_1)=\{h_1,h_3\}$, $G_2(\neg e_2)=\{h_1,h_3\}$

$F_3(e_1)=\{h_1,h_2\}$, $F_3(e_2)=\{h_1,h_2\}$ and $G_3(\neg e_1)=\{h_3\}$, $G_3(\neg e_2)=\{h_3\}$.

Then $(U,\tau,E,\neg E)$ is a bipolar soft connected space.

Now take $Y=\{h_1,h_2\}$. Then $\tau_Y=\{(\Phi,\tilde{Y},E),(\tilde{Y},\Theta,E),({}^YF_1,{}^YG_1,E),({}^YF_2,{}^YG_2,E),({}^YF_3,{}^YG_3,E)\}$, where

${}^YF_1(e_1)=\{h_1\}$, ${}^YF_1(e_2)=\{h_1\}$ and ${}^YG_1(\neg e_1)=\{h_2\}$, ${}^YG_1(\neg e_2)=\{h_2\}$,

${}^YF_2(e_1)=\{h_2\}$, ${}^YF_2(e_2)=\{h_2\}$ and ${}^YG_2(\neg e_1)=\{h_1\}$, ${}^YG_2(\neg e_2)=\{h_1\}$,

${}^YF_3(e_1)=Y$, ${}^YF_3(e_2)=Y$ and ${}^YG_3(\neg e_1)=\varphi$, ${}^YG_3(\neg e_2)=\varphi$. Then $(Y,\tau_Y,E,\neg E)$ is a bipolar soft disconnected subspace of bipolar soft connected space.

Example 8. Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$ and $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E)\}$ where

$F_1(e_1)=\{h_1\}$, $F_1(e_2)=\{h_2\}$ and $G_1(\neg e_1)=\{h_2\}$, $G_1(\neg e_2)=\{h_1,h_3\}$,

$F_2(e_1)=\{h_2,h_3\}$, $F_2(e_2)=\{h_1,h_3\}$ and $G_2(\neg e_1)=\varphi$, $G_2(\neg e_2)=\{h_2\}$,

$F_3(e_1)=\varphi$, $F_3(e_2)=\varphi$ and $G_3(\neg e_1)=\{h_2\}$, $G_3(\neg e_2)=U$.

Then $(U,\tau,E,\neg E)$ is a bipolar soft disconnected space.

Now take $Y=\{h_3\}$. Then $\tau_Y=\{(\Phi,\tilde{Y},E),(\tilde{Y},\Theta,E),({}^YF_1,{}^YG_1,E),({}^YF_2,{}^YG_2,E),({}^YF_3,{}^YG_3,E)\}$ where

${}^YF_1(e_1)=\varphi$, ${}^YF_1(e_2)=\varphi$ and ${}^YG_1(\neg e_1)=\varphi$, ${}^YG_1(\neg e_2)=Y$,

${}^YF_2(e_1)=Y$, ${}^YF_2(e_2)=Y$ and ${}^YG_2(\neg e_1)=\varphi$, ${}^YG_2(\neg e_2)=\varphi$,

${}^YF_3(e_1)=\varphi$, ${}^YF_3(e_2)=\varphi$ and ${}^YG_3(\neg e_1)=\varphi$, ${}^YG_3(\neg e_2)=Y$. Then $(Y,\tau_Y,E,\neg E)$ is a bipolar soft connected subspace of bipolar soft disconnected space.

4. Bipolar Soft Compact Spaces

In this section, we study another important property of bipolar soft topological spaces called the bipolar soft compactness. Bipolar soft compact spaces are investigated and some results related to this concept are derived.

Definition 24. A family $\Psi=\{(F_\alpha,G_\alpha,E)\}_{\alpha\in J}$ of bipolar soft sets is called the bipolar soft cover of a bipolar soft set (F,G,E) if $(F,G,E)\subseteq\tilde{\cup}_{\alpha\in J}(F_\alpha,G_\alpha,E)$.

Further, it is called the bipolar soft open cover of a bipolar soft set (F,G,E) if each member of Ψ is a bipolar soft open set. A bipolar soft subcover of Ψ is a subfamily of Ψ which is also a bipolar soft cover.

Definition 25. A bipolar soft topological space $(U,\tau,E,\neg E)$ is called a bipolar soft compact space, if each bipolar soft open cover of (υ,Θ,E) has a finite bipolar soft subcover.

Example 9. Let $U=\{h_1,h_2,h_3\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$, and $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E),(F_7,G_7,E)\}$ where $(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E),(F_7,G_7,E)$ are bipolar soft sets over U defined as follows:

$F_1(e_1)=\{h_2\}$, $F_1(e_2)=\{h_2\}$ and $G_1(\neg e_1)=\{h_3\}$, $G_1(\neg e_2)=\{h_3\}$,

$F_2(e_1)=\{h_1\}$, $F_2(e_2)=\{h_1\}$ and $G_2(\neg e_1)=\{h_3\}$, $G_2(\neg e_2)=\{h_3\}$,

$F_3(e_1)=\{h_1,h_2\}$, $F_3(e_2)=\{h_1,h_2\}$ and $G_3(\neg e_1)=\{h_3\}$, $G_3(\neg e_2)=\{h_3\}$,

$F_4(e_1)=\{h_2,h_3\}$, $F_4(e_2)=\{h_2,h_3\}$ and $G_4(\neg e_1)=\varphi$, $G_4(\neg e_2)=\varphi$,

$F_5(e_1)=\{h_1,h_3\}$, $F_5(e_2)=\{h_1,h_3\}$ and $G_5(\neg e_1)=\varphi$, $G_5(\neg e_2)=\varphi$,

$F_6(e_1) = \varphi$, $F_6(e_2) = \varphi$ and $G_6(\neg e_1) = \{h_3\}$, $G_6(\neg e_2) = \{h_3\}$,

$F_7(e_1) = \{h_3\}$, $F_7(e_2) = \{h_3\}$ and $G_7(\neg e_1) = \varphi$, $G_7(\neg e_2) = \varphi$. Then $(U, \tau, E, \neg E)$ is a bipolar soft topological space over *U*. Further, we can easily observe that $(U, \tau, E, \neg E)$ is a bipolar soft compact space because every open cover of (υ, Θ, E) has a finite subcover.

Example 10. Let $U = \mathrm{N}$ be the universe set of natural numbers, let $E = \{e_1, e_2\}$ and $\neg E = \{\neg e_1, \neg e_2\}$ be the set of parameters and the not set of parameters, respectively. Let τ be the bipolar soft topology over N, consisting of all bipolar soft sets defined on the parameter set *E*, generated by the bipolar soft sets (F_1, G_1, E), (F_2, G_2, E), (F_3, G_3, E), ..., where

$F_1(e_1) = \{1,2\}$, $F_1(e_2) = \{1,2\}$ and $G_1(\neg e_1) = U \setminus \{1,2\}$, $G_1(\neg e_2) = U \setminus \{1,2\}$,

$F_2(e_1) = \{2,3\}$, $F_2(e_2) = \{2,3\}$ and $G_2(\neg e_1) = U \setminus \{2,3\}$, $G_2(\neg e_2) = U \setminus \{2,3\}$,

$F_3(e_1) = \{3,4\}$, $F_3(e_2) = \{3,4\}$ and $G_3(\neg e_1) = U \setminus \{3,4\}$, $G_3(\neg e_2) = U \setminus \{3,4\}$, ...,

$F_n(e_1) = \{n, n+1\}$, $F_n(e_2) = \{n, n+1\}$ and $G_n(\neg e_1) = U \setminus \{n, n+1\}$, $G_n(\neg e_2) = U \setminus \{n, n+1\}$

.....

Then the bipolar soft topological space $(U, \tau_1, E, \neg E)$ over *U* generated by the bipolar soft sets $\{(F_n, G_n, E) : n \in \mathrm{N}\}$ is not a bipolar soft compact space since $\Psi = \{(F_n, G_n, E) : n \in \mathrm{N}\}$ is a bipolar soft open cover of N with no finite bipolar soft subcover.

Definition 26. Let $(U, \tau_1, E, \neg E)$ and $(U, \tau_2, E, \neg E)$ be two bipolar soft topological spaces over the universe *U*. If $\tau_1 \subseteq \tau_2$, then τ_2 is said to be finer than τ_1. If $\tau_1 \subseteq \tau_2$ or $\tau_2 \subseteq \tau_1$, then τ_1 is comparable with τ_2.

Proposition 9. Let $(U, \tau_2, E, \neg E)$ be a bipolar soft compact space and $\tau_1 \subseteq \tau_2$. Then $(U, \tau_1, E, \neg E)$ is a bipolar soft compact.

Proof. Let $\{(F_\alpha, G_\alpha, E)\}_{\alpha \in J}$ be the bipolar soft open cover of (υ, Θ, E) in $(U, \tau_1, E, \neg E)$. Since $\tau_1 \subseteq \tau_2$, then $\{(F_\alpha, G_\alpha, E)\}_{\alpha \in J}$ is the bipolar soft open cover of (υ, Θ, E) by bipolar soft open sets of $(U, \tau_2, E, \neg E)$. But $(U, \tau_2, E, \neg E)$ is a bipolar soft compact space.

Therefore $(U, \Theta, E) \tilde{\subseteq} (F_{\alpha_1}, G_{\alpha_1}, E) \tilde{\cup} (F_{\alpha_2}, G_{\alpha_2}, E) \ldots \tilde{\cup} (F_{\alpha_n}, G_{\alpha_n}, E)$, for some $\alpha_1, \alpha_2, ..., \alpha_n \in J$. Hence $(U, \tau_1, E, \neg E)$ is a bipolar soft compact space.

Theorem 6. Let $(Y, \tau_Y, E, \neg E)$ be a bipolar soft subspace of $(U, \tau, E, \neg E)$. Then $(Y, \tau_Y, E, \neg E)$ is a bipolar soft compact space if and only if every cover of $(\tilde{Y}, \Theta, E)$ by bipolar soft open sets in *U* contains a finite subcover.

Proof. Let $(Y, \tau_Y, E, \neg E)$ be a bipolar soft compact space and $\{(F_\alpha, G_\alpha, E)\}_{\alpha \in J}$ be a cover of $(\tilde{Y}, \Theta, E)$ by bipolar soft open sets in *U*. Now, $Y \subseteq \cup_{\alpha \in J} (Y \cap F_\alpha(e))$ for each $e \in E$, and $? \subseteq (Y \cap G_\alpha(\neg e))$ for each $\neg e \in \neg E$.

Therefore, $\{({}^Y F_\alpha, {}^Y G_\alpha, E)\}_{\alpha \in J}$ is a bipolar soft open cover of (Y, Θ, E).

Since $(Y, \tau_Y, E, \neg E)$ is a bipolar soft compact space, therefore, we have

$(\tilde{Y}, \Theta, E) \tilde{\subseteq} ({}^Y F_{\alpha_1}, {}^Y G_{\alpha_1}, E) \tilde{\cup} ({}^Y F_{\alpha_2}, {}^Y G_{\alpha_2}, E) \tilde{\cup} \ldots \tilde{\cup} ({}^Y F_{\alpha_n}, {}^Y G_{\alpha_n}, E)$ for some $\alpha_1, \alpha_2, \cdots, \alpha_n \in J$.

This implies that $\{(F_{\alpha_i}, G_{\alpha_i}, E)\}_{i=1}^n$ is a bipolar subcover of (Y, Θ, E) by bipolar soft open sets in *U*.

Conversely, suppose $\{({}^Y F_\alpha, {}^Y G_\alpha, E)\}_{\alpha \in J}$ is a bipolar soft open cover of (Y, Θ, E). It is easy to see that $\{(F_\alpha, G_\alpha, E)\}_{\alpha \in J}$ is a bipolar soft open cover of (Y, Θ, E) by bipolar soft open sets in *U*. Therefore, by given hypothesis we have $(\tilde{Y}, \Theta, E) \tilde{\subseteq} (F_{\alpha_1}, G_{\alpha_1}, E) \tilde{\cup} (F_{\alpha_2}, G_{\alpha_2}, E) \tilde{\cup} \ldots \tilde{\cup} (F_{\alpha_n}, G_{\alpha_n}, E)$ for some $\alpha_1, \alpha_2, \cdots, \alpha_n \in J$. Thus, $\{({}^Y F_{\alpha_i}, {}^Y G_{\alpha_i}, E)\}_{i=1}^n$

is a bipolar subcover of (Y,Θ,E). Hence $(Y,\tau_Y,E,\neg E)$ is a bipolar soft compact space.

Definition 27. Let $(U,\tau,E,\neg E)$ be a bipolar soft topological space over U and let $\beta \subseteq \tau$. If every element of τ can be written as the union of the elements of β, then β is called a bipolar soft basis for bipolar soft topology τ. Each element of β is called a bipolar soft basis element.

Example 11. Let $U=\{h_1,h_2,h_3,h_4\}$, $E=\{e_1,e_2\}$, $\neg E=\{\neg e_1,\neg e_2\}$, and $\tau=\{(\Phi,\tilde{u},E),(\upsilon,\Theta,E),(F_1,G_1,E),(F_2,G_2,E),(F_3,G_3,E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E),(F_7,G_7,E),(F_8,G_8,E)\}$ where $(F_2,G_2,E),(F_3,G_3,E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E),(F_7,G_7,E),(F_8,G_8,E)$ are bipolar soft sets over U defined as follows:

$F_1(e_1)=\{h_1,h_2,h_3\}$, $F_1(e_2)=\{h_1,h_2\}$ and $G_1(\neg e_1)=\{h_4\}$, $G_1(\neg e_2)=\{h_3,h_4\}$,

$F_2(e_1)=\{h_1,h_2,h_4\}$, $F_2(e_2)=\{h_1,h_2\}$ and $G_2(\neg e_1)=\{h_3\}$, $G_2(\neg e_2)=\{h_3,h_4\}$,

$F_3(e_1)=\{h_3,h_4\}$, $F_3(e_2)=\varphi$ and $G_3(\neg e_1)=\{h_1,h_2\}$, $G_3(\neg e_2)=U$,

$F_4(e_1)=\{h_1,h_2\}$, $F_4(e_2)=\{h_1,h_2\}$ and $G_4(\neg e_1)=\{h_3,h_4\}$, $G_4(\neg e_2)=\{h_3,h_4\}$,

$F_5(e_1)=\{h_3\}$, $F_5(e_2)=\varphi$ and $G_5(\neg e_1)=\{h_1,h_2,h_4\}$, $G_5(\neg e_2)=U$,

$F_6(e_1)=\{h_4\}$, $F_6(e_2)=\varphi$ and $G_6(\neg e_1)=\{h_1,h_2,h_3\}$, $G_6(\neg e_2)=U$,

$F_7(e_1)=U$, $F_7(e_2)=\{h_1,h_2\}$ and $G_7(\neg e_1)=\varphi$, $G_7(\neg e_2)=\{h_3,h_4\}$,

$F_8(e_1)=\{h_3,h_4\}$, $F_8(e_2)=\{h_3,h_4\}$ and $G_8(\neg e_1)=\{h_1,h_2\}$, $G_8(\neg e_2)=\{h_1,h_2\}$. Then $(U,\tau,E,\neg E)$ is a bipolar soft topological space over U. Now if we take $\beta=\{(\Phi,\tilde{u},E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E),(F_7,G_7,E),(F_8,G_8,E)\}$, then β is a bipolar soft basis for τ.

Next, if we take $\beta' \subseteq \tau$ where $\beta'=\{(\Phi,\tilde{u},E),(F_1,G_1,E),(F_4,G_4,E),(F_5,G_5,E),(F_6,G_6,E)\}$ then β' is not a bipolar soft basis because (υ,Θ,E) cannot be written as the union of the elements of β'.

Theorem 7. A bipolar soft topological space $(U,\tau,E,\neg E)$ is a bipolar soft compact space if and only if there is a bipolar soft basis β for τ such that every bipolar soft cover of (υ,Θ,E) by the elements of β has a finite bipolar soft sub-cover.

Proof. Let $(U,\tau,E,\neg E)$ be a bipolar soft compact space. Obviously τ is a bipolar soft basis for τ. Therefore, every bipolar soft cover of (υ,Θ,E) by elements of τ has a finite bipolar soft subcover.

Conversely, to show $(U,\tau,E,\neg E)$ is a bipolar soft compact, let $\{(L_\alpha,M_\alpha,E)\}_{\alpha\in J}$ be a bipolar soft open cover of (υ,Θ,E). We can write (L_α,M_α,E) as a union of basis element for each $\alpha \in J$. These elements form a bipolar soft open cover of (υ,Θ,E) such that $\left\{(F_\beta,G_\beta,E)\right\}_{\beta\in I}$. Now, by given hypothesis, for some $\beta_1,\beta_2,\dots\beta_n \in I$, we have

$U=F_{\beta_1}(e)\cup F_{\beta_2}(e)\cup F_{\beta_3}(e)\cdots\cup F_{\beta_n}(e)$, for each $e\in E$ and

$\varphi=G_{\beta_1}(\neg e)\cap G_{\beta_2}(\neg e)\cap\cdots\cap G_{\beta_n}(\neg e)$ for each $\neg e\in\neg E$. That is $(\upsilon,\Theta,E)=(F_{\beta_1},G_{\beta_1},E)\tilde{\cup}(F_{\beta_2},G_{\beta_2},E)\tilde{\cup}\dots\tilde{\cup}(F_{\beta_n},G_{\beta_n},E)$ for some $\beta_1,\beta_2,\dots\beta_n\in I$. Now, $(F_{\beta_i},G_{\beta_i},E)\tilde{\subseteq}(L_{\alpha_i},M_{\alpha_i},E)$, for each $1\le i\le n$. This implies that $\left\{(L_{\alpha_i},M_{\alpha_i},E)\right\}_{i=1}^{n}$ is a finite bipolar subcover of (υ,Θ,E). Hence $(U,\tau,E,\neg E)$ is a bipolar soft compact space.

5. Conclusions

During the study we have gone into detail about defining and finding out the properties of bipolar soft connected spaces, bipolar soft disconnected spaces and bipolar soft compact spaces. The credit of strengthening the foundations in the tool box of bipolar soft topology will be given to these newly defined concepts. The findings and results which we have drawn can be applied to solve existing problems in various fields which contain uncertainty.

References

Aktas, H., & Cagman, N. (2007). Soft sets and soft groups. *Information Sciences, 177*, 2726-2735.

Ali, M. I., & Shabir, M. (2010). Comments on De Morgan's law in fuzzy soft sets. *The Journal of Fuzzy Mathematics, 18*, 679-686

Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers and Mathematics with Applications, 61*, 2647-2654.

Ali, M. I., Feng, F., Liu, X. Y., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers and Mathematics with Applications, 57*, 1547-1553.

Aygunoglu, A., & Aygun, H. (2012). Some notes on soft topological spaces. *Neural Computing and Applications, 21*, 113-119.

Cagman, N., Karatas, S., & Enginoglu, S. (2011). Soft topology. *Computers and Mathematics with Applications, 62*, 351-358.

Gau, W. L., & Buehrer, D. J. (1993). Vague sets. *IEEE Transactions on Systems, Man, and Cybernetics, 23*(2), 610-614.

Hayat, K., & Mahmood, T. (2015). Some applications of bipolar soft set: Characterizations of two isomorphic Hemi-Rings via BSI-h-Ideals. *British Journal of Mathematics and Computer Science, 13*, 1-21.

Hussain, S. (2014). A note on soft connectedness. *Journal of the Egyptian Mathematical Society, 23*, 6-11.

Hussain, S., & Ahmad, B. (2011). Some properties of soft topological spaces. *Computers and Mathematics with Applications, 62*, 4058- 4067.

Jun, Y. B. (2008). Soft BCK/BCI-algebras. *Computers and Mathematics with Applications, 56*, 1408-1413.

Jun,Y. B., & Park, C. H. (2008). Applications of soft sets in ideal theory of BCK/BCI-Algebras. *Information Sciences, 178*, 2466-2475.

Karaaslan, F., & Karatas, S. (2015). A new approach to bipolar soft sets and its applications. *Discrete Mathematics, Algorithms and Applications, 7*(4). doi:10.1142/S1793830915500548

Khalil, O. H., & Ghareeb, A. (2015). Spatial object modeling in soft topology. *Songklanakarin Journal of Science and Technology, 37*(4), 493-498.

Lin, F. (2013). Soft connected spaces and soft paracompact spaces. *International Journal of Mathematical Science and Engineering, 7*(2), 1-7.

Maji, P. K., Biswas, R., & Roy, R. (2003). Soft set theory. *Computers and Mathematics with Applications, 45*, 555-562.

Min, W. K. (2011). A note on soft topological spaces. *Computers and Mathematics with Applications, 62*, 3524-3528.

Molodtsov, D. (1999). Soft set theory first results. *Computers and Mathematics with Applications, 37*, 19-31.

Pawlak, Z. (1982). Rough sets. *International Journal of Information and Computer Science, 11*, 341-356.

Peyghan, E., Samadi, B., & Tayebi, A. (2014). Some results related to soft topological spaces. *Facta Universitatis, Series: Mathematics and Informatics, 29*(4), 325-336. Retrieved from http://casopisi.junis.ni.ac.rs/index.php/FUMathInf/article/view/124/pdf

Peyghan, E., Samadi, B., & Tayebi, A. (2013). About soft topological spaces. *Journal of New Results in Science, 2*, 60-75.

Shabir, M., & Ali, M. I. (2009). Soft ideals and generalized fuzzy ideals in semigroups. *New Mathematics and Natural Computation, 5*, 599-615.

Shabir, M., & Naz, M. (2011). On soft topological spaces. *Computers and Mathematics with Applications, 61*, 1786-1799.

Shabir, M., & Naz, M. (2013). On bipolar soft sets. Retrieved from https://arxiv.org/abs/1303.1344

Varol, B. P., Shostak, A., & Aygun, H. (2012). A new approach to soft topology. *Hacettepe Journal of Mathematics and Statistics, 41*, 15-24.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control, 8*, 338-353.

Zakari, A. H., Ghareeb, A., & Omran, S. (2016). On soft weak structures. Soft Computing. doi:10.1007/s00500-016-2136-8.

Zorlutuna, I., Akdag, M., Min, W.K., & Atmaca, S. (2012). Remarks on soft topological spaces. *Annals of Fuzzy Mathematics and Informatics, 3*, 171-185.