



**An artificial bee colony algorithm for the vehicle routing problem with backhauls and time windows**

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## Original Article

### An artificial bee colony algorithm for the vehicle routing problem with backhauls and time windows

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#### Abstract

The vehicle routing problem with backhauls and time windows (VRPBTW) aims to find a feasible vehicle route that minimizes the total traveling distance while imposing capacity, backhaul, and time-window constraints. We present an enhanced artificial bee colony algorithm (EABCA), which is a meta-heuristic, to solve this problem. Three strategies—a forbidden list, the sequential search for onlookers, and the combination of 1-move intra-route exchange and  $\lambda$ -interchange technique—are introduced for EABCA. The proposed method was tested on a set of benchmark instances. The computational results show that the EABCA can produce better solutions than the basic ABCA, and it discovered many new best-known solutions.

**Keywords:** meta-heuristic, artificial bee colony, backhaul, time window, vehicle routing problems

#### 1. Introduction

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4 The vehicle routing problem with backhaul and time window (VRPBTW) is  
5 extended from the vehicle routing problem with backhaul (VRPB) by adding a specified  
6 service time window for each customer. There are three main constraint categories for  
7 VRPBTW model, namely capacity constraints, backhaul constraints and time window  
8 constraints. For the capacity constraints, the number of customers serviced by a vehicle  
9 is restricted by the capacity of the vehicle. For the backhaul constraints, the vehicles  
10 serve all demands of the linehaul customers and the same vehicles also pick up demands  
11 from the backhaul customers. In addition, the backhaul customers cannot be served  
12 before linehaul customers. For the time window constraints, the vehicle arrival time at  
13 each customer must not exceed the upper bound of the customer's time window. In  
14 general, the VRPBTW is a class of the NP-hard combinatorial optimization problems,  
15 which is too difficult to solve exactly within a reasonable time. Consequently, there are  
16 many heuristic methods proposed to get a near optimal solution for this problem.  
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33 An increasing number of the publications on heuristic approaches for vehicle  
34 routing problem have been developed for the past two decades. However, only few  
35 studies have been devoted to the VRPBTW. A brief review of these studies is divided  
36 into two parts based on the types of the proposed methods, namely non-meta-heuristic  
37 methods and meta-heuristic methods.  
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45 A few non-meta-heuristic methods were proposed to solve VRPBTW. Thangiah  
46 et al. (1996) presented a push forward insert heuristic (PFIH). This algorithm applied an  
47 insertion heuristic for route construction and improved solution by  $\lambda$ -interchange and 2-  
48 opt\* exchange procedures to solve VRPBTW problems. The algorithm was tested on  
49 benchmark instances of G elinas et al. (1995). Although the solutions of PFIH were  
50 within 2.5% of the optimum on average, PFIH almost always gave worse results than  
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4 average for large-sized problems. Ropke and Pisinger (2006) transformed the VRPBTW  
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6 into the VRPB by ordering the routes according to time window constraints and then  
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8 used a neighbor search algorithm to solve the problem. Although the unified heuristic  
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10 could improve the best-known solution for many instances and decrease the necessary  
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12 number of vehicles, the computational time increases considerably with the problem  
13  
14 size. Worawattavechai et al. (2016a) presented a nearest urgent candidate heuristic  
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16 (NUC) for the VRPBTW. This algorithm starts by sorting the customers according to  
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18 the urgency of delivery before adding into the route by considering their closeness while  
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20 a candidate list technique is used to enforce the urgency order. Although NUC heuristic  
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22 performed better than the PFIH, it underperformed the existing algorithm for the large-  
23  
24 sized problems. In general, non-meta-heuristic methods can achieve good optimization  
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26 results quickly while it is relatively simple and easy to apply to the problems. However,  
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28 the efficacy of these methods decreases when the problem size becomes large.  
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34 There are more studies that focused on meta-heuristic methods for VRPBTW.  
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36 Potvin et al. (1996b) described a genetic algorithm (GA) coupled with a greedy  
37  
38 insertion heuristic to find a good insertion order of the customers. The results showed  
39  
40 that GA solutions were within 1% of the optimum on average by using benchmark  
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42 instances of Gélinas et al. (1995). However, it was very expensive in terms of  
43  
44 computational resources, and it was frequently prone to premature converging. Cho and  
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46 Wang (2005) presented a meta-heuristic which is based on an acceptable threshold  
47  
48 combined with modified nearest neighbor and exchange procedures for solving the  
49  
50 VRPBTW. Although this method could decrease the computational time, it  
51  
52 underperformed the GA proposed by Potvin et al. (1996b). Küçüköglu and Öztürk  
53  
54 (2014) introduced a differential evolution algorithm (DEA) for VRPBTW and applied it  
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4 for a catering firm. DEA was tested with several benchmark problems. The results  
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6 showed that this algorithm could obtain some new best-known solutions. Since DEA  
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8 was originally designed for continuous problems, it is hard to find a good encoding  
9  
10 procedure to adapt DEA to integer problems like VRPBTW. Later, Küçükoğlu and  
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12 Öztürk (2015) proposed an advanced hybrid meta-heuristic algorithm (HMA), which  
13  
14 combined tabu search algorithm and simulated annealing algorithm to obtain more  
15  
16 effective solutions for the VRPBTW. The results indicated that the HMA performed  
17  
18 better than the DEA. However, one of the disadvantages of the hybrid algorithm was  
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20 that it took a lot of computational time. Worawattawechai et al. (2016b) proposed the  
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22 nearest neighbor with roulette wheel selection method (NNRW) as an initial solution  
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24 algorithm for the cuckoo search (CS) algorithm. The result reported that the CS  
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26 algorithm could produce better solutions than the best-known solutions for the majority  
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28 of small- and medium-sized instances. However, it did not perform as well for large  
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30 problems.  
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36 Artificial bee colony algorithm (ABCA) is another meta-heuristic method that  
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38 has been applied to VRP. It was first introduced by Karaboga (2005). It is firstly applied  
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40 to the capacitated vehicle routing problems (CVRP) by Szeto et al. (2011) with some  
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42 enhancements. The results show that the enhanced version of ABCA outperformed the  
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44 original one, and it could produce good solutions when compared with the existing  
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46 heuristics. Alzaqebah et al. (2016) presented the modified artificial bee colony for the  
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48 vehicle routing problems with time windows (VRPTW). In this study, the list of  
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50 abandoned solutions was used to generate new solutions. The results showed that the  
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52 modified ABCA obtained good results when compared with the best-known results. An  
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54 improved artificial bee colony algorithm for a real case in Dalian was introduced by Yu  
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4 et al. (2016). In this version of ABCA, three strategies were applied, namely an adaptive  
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6 strategy, a crossover operation, and a mutation operation. The results showed that some  
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8 of solutions were better than the best-known solution when tested on benchmark  
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10 problems of Solomon (1987) for VRPTW.  
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14 There are many reasons that motivate the authors to use ABCA to solve  
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16 VRPBTW in this paper. Firstly, ABCA was successfully applied to VRP and VRPTW  
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18 as described in the above paragraph. Secondly, ABCA is a meta-heuristic, which means  
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20 the exploring area of the solution space is larger than non-meta-heuristics (PFIH,  
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22 unified heuristic, NUC heuristic). Thus, it can achieve good optimization results,  
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24 especially in the large-sized problem. Thirdly, ABCA is a population-based heuristic  
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26 which starts with a number of initial solutions. Therefore, it can explore more in the  
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28 solution space and get more chance to obtain the better solutions than non-population-  
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30 based heuristic (e.g. HMA). Moreover, a population-based heuristic can be enhanced  
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32 with parallel computing or distributed computing. Finally, ABCA can prevent the  
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34 search from premature convergence problem which is the weakness of other population-  
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36 based heuristics (e.g. GA and DEA). This is because, in the scout bee stage, the stalled  
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38 solutions are removed from the population and a new random generated solution is  
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40 added to the population. This process also amplifies global search capability.  
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45 There are a few studies (Tuntitippawan and Asawarungsaengkul 2016a, 2016b)  
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47 that apply ABCA for solving VRPBTW. Tuntitippawan and Asawarungsaengkul  
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49 (2016a) applied ABCA to small and medium problems and Tuntitippawan and  
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51 Asawarungsaengkul (2016b) applied ABCA to small, medium, and large problems.  
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53 However, the computational results showed that it still underperformed the existing  
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55 heuristics in many instances, especially in the large-scale problems. It is necessary to  
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4 extend the exploration on the solution space or, equivalently, to expand the capability of  
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6 the neighborhood search. Therefore, we introduce the enhanced artificial bee colony  
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8 algorithm (EABCA) by applying a forbidden list strategy to prevent duplicated initial  
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10 solutions (which initially extends the exploration on the solution space), the sequential  
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12 search strategy for onlookers to explore the neighborhood near the high-quality food  
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14 source, and the intra-route and inter-route exchange combination strategy to obtain the  
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16 better solutions. Moreover, the parametrization is studied in this paper.  
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## 20 21 22 **2. Enhanced Artificial Bee Colony Algorithm for VRPBTW**

### 23 24 *2.1 The General Concept of an Artificial Bee Colony*

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26 The artificial bee colony is inspired by the intelligent finding food sources  
27  
28 behavior of the honey bees around the hives proposed by Karaboga (2005). A colony of  
29  
30 the bees consists of three types of bees: employed bees, onlookers and scouts. The  
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32 employed bees search for available nectar sources and share this information with the  
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34 onlookers via a waggle dance at the dancing area. The onlookers select the food sources  
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36 by evaluating quality of nectar sources from the waggle dance to be further explored.  
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38 When the quality of food sources is not improved within a time limit, the employed  
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40 bees abandon the food source and turn into scout bees to find new food sources.  
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44 The ABCA starts by generating a number of nectar sources (initial solutions)  
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46 and assigning an employed bee to each food source. Each employed bee explores new  
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48 food source near its original food source (neighborhood search) and measures the nectar  
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50 amounts (fitness value). If the new source has more nectar, it will replace the old one.  
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52 Then the employed bees return to the hive with the information of the updated food  
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54 sources, which is shared with the onlookers by the waggle dance. Each onlooker selects  
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4 a food source with a probability that depends on the nectar amounts (the roulette wheel  
5 method). In particular, a food source with higher nectar amounts has a higher  
6 probability to be selected by an onlooker than ones with lower nectar amounts. Then  
7 each onlooker finds a new food source around the selected food sources (neighborhood  
8 search) and evaluates the amount of nectar. The employed bee will abandon its old food  
9 source and go to the new one if it has more nectar. In the case that the quality of food  
10 source is not improved within a time limit, the employed bee will also abandon the old  
11 food source and become a scout bee that searches for the new food source by randomly  
12 generating a new solution. After the scout bee finds a new food source, it becomes an  
13 employed bee again. This process will repeat until a stopping criterion is reached.

## 24 25 26 *2.2 Main Steps*

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28 The steps of the EABCA for solving the VRPBTW model can be described as  
29 follows:  
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33 Step 1 Generate a set of initial solutions (food sources) by the nearest neighbor  
34 with roulette wheel selection method. The forbidden list strategy is also  
35 applied in this process. (Details in Section 2.3) Then assign each food  
36 source to each employed bee.  
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43 Step 2 Evaluate the fitness of each solution and record the global best solution.  
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46 Step 3 Apply the neighborhood search on each food source. An employed bee  
47 abandons its old food source if a new one with better fitness is found.  
48 Otherwise, increment the time limit counter of the food source.  
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52 Step 4 For each onlooker, select a food source by using the fitness-based  
53 roulette wheel selection method and improve the food source by the  
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neighborhood search. If the onlooker bee finds a new neighbor solution with better fitness, the employed bee associated with the food source abandons its old food source and go to the new one.

Step 5 Update the global best solution if a solution has better fitness than the current best one.

Step 6 Check the time limit counter of each food source. If it reaches the predetermined number, the food source is replaced by a new randomly generated solution.

Step 7 If the number of iterations reaches the maximum, then the algorithm finishes. Otherwise, go back to Step 3.

### 2.3 Initial Solution Construction

Küçüköğlu and Öztürk (2015) proposed an improved nearest neighbor heuristic for constructing an initial solution for VRPBTW. They computed the closeness of customer  $i$  to customer  $j$  by using  $proximity_{ij}$ , which is defined as:  $proximity_{ij} = \alpha c_{ij} + \beta h_{ij} + \gamma v_{ij}$ , where  $\alpha, \beta, \gamma$  denote weight parameters such that  $\alpha + \beta + \gamma = 1, \alpha \geq 0, \beta \geq 0, \gamma \geq 0$ ;  $c_{ij}$  denotes the traveling time from customer  $i$  to customer  $j$ ;  $h_{ij}$  denotes the idle time before servicing customer  $j$  after customer  $i$ ; and  $v_{ij}$  denotes the urgency of delivery to customer  $j$  after customer  $i$  expressed as the time remaining until the vehicle's last possible service start for customer  $j$ . Then the closeness of customers  $i$  and  $j$ , denoted by  $closeness_{ij}$ , is defined as the reciprocal of  $proximity_{ij}$ .

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4 This paper adopts the above definition of closeness and uses it in the  
5 construction of the initial solutions. Each initial solution is constructed by the nearest  
6 neighbor with roulette wheel selection method proposed by Worawattavechai et al.  
7 (2016b). An initial solution construction always starts a tour with the depot, and then  
8 finds the next customer by spinning the roulette wheel. If the next customer violates the  
9 constraints, we spin the roulette wheel again to find a new one. If we cannot find the  
10 next customer without violating constraints, we end this tour and begin a new tour. This  
11 process is repeated until all customers are served.  
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22 Using the roulette wheel method alone can cause duplicate initial solutions,  
23 which hinders the exploration space down the road. In EABCA, the forbidden list  
24 strategy is applied to prevent this problem. Initially, the forbidden list is empty.  
25 Subsequently, after a new feasible initial solution is obtained, the solution will be  
26 checked with the forbidden list. If the solution is not in the list, then it is added to the  
27 forbidden list. Otherwise, the solution will be abandoned. The process is repeated until  
28 the number of solutions in the forbidden list reaches to the specified number of initial  
29 solutions.  
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#### 40 *2.4 Fitness Function*

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42 In this paper, we compute the Euclidean distance between customer  $i$  and  
43 customer  $j$  using the following formula (Kohl et al., 1997 and Kohl et al., 1999):  
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$$48 \quad c_{ij} = \frac{10 \left( \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \right)}{10}$$

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4 where  $(x_i, y_i)$  is the coordinate of customer  $i$  and  $(x_j, y_j)$  is the coordinate of customer  
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7  $j$ . The traveling time between two customers is assumed to be the same as the distance  
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9 between them.

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12 The fitness value ( $f$ ) of a solution is a reciprocal of the total distance traveled by  
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14 all vehicles in the solution.

### 15 16 17 18 *2.5 Neighborhood Search*

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20 The local search in the ABCA proposed by Tuntitippawan and  
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22 Asawarungsaengkul (2016b) only uses the  $\lambda$ -interchange, which is an inter-route  
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24 operator that considers two routes at once. To extend the search ability, the EABCA can  
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26 either randomly apply  $\lambda$ -interchange or 1-move intra-route exchange, which work on a  
27  
28 single route, for its neighborhood search. Since the 1-move operator improves the  
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30 solution by deleting a customer and then inserting it into the same route, it helps  
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32 rearranging the customer in the route. The experimental parameter testing discussed in  
33  
34 Section 3.1 indicates that this setting gives better solution than using  $\lambda$ -interchange  
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36 alone (See Figure 5). The details of both types of operator are explained below.

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38  
39 The 1-move intra-route exchange is a practical operator for the traveling  
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41 salesman problem. We adapt this operator to improve each route in the vehicle routing  
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43 solution by removing one customer from a route and insert it back to the same route in a  
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45 different position. An example of the 1-move intra-route exchange is given in Figure 1.

#### 46 47 48 **Figure 1 An example of the 1-move intra-route exchange.**

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53  $\lambda$ -interchange is a generalization of a relocation operator for the vehicle routing  
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55 problem proposed by Osman (1993). The idea is to exchange a subset of customers of  
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4 size  $x$  with a subset of customers of size  $y$  from a different route, which can be  
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6 represented by the operator  $(x, y)$  where  $x$  and  $y$  are nonnegative integers not bigger  
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8 than  $\lambda$ . In this paper, we use  $\lambda = 4$ . For examples, one possible operator is  $(1, 0)$  which  
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10 means moving one customer in the one route to another route as shown in Figure 2.  
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12 Another possible operator is  $(2, 1)$  which means exchanging two customers in the one  
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14 route with one customer in another route as shown in Figure 3.  
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17 **Figure 2 An example of the operation  $(1, 0)$ .**

18 **Figure 3 An example of the operation  $(2, 1)$ .**

#### 23 *2.6 Fitness-Based Roulette Wheel Selection method*

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25 The fitness-based roulette wheel selection method is applied in the food source  
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27 selection process. In this method, each onlooker selects a food source to explore  
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29 according to the probability that depends on the nectar amount (or fitness value). After  
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31 selecting a food source, each onlooker finds a new food source around the selected food  
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33 source and evaluates the amount of nectar.  
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38 In the original version, if there are many onlooker bees selecting the same food  
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40 source, each onlooker individually searches for a new food source and the old food  
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42 source is replaced by the best of those new food sources. In our enhanced version, if  
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44 there are many onlooker bees selecting the same food source, they will be queued up for  
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46 searching a new food source. The search can only be performed by one onlooker bee at  
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48 the time. If the previous onlooker bee finds a new better food source, the next onlooker  
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50 bee will start from the newly found food source and look for a better one. Otherwise,  
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52 the next onlooker bee will start from the same food source as the previous one. In this  
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4 way, algorithm will be given opportunities to be further explored in good regions of the  
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6 solution.  
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### 10 11 **3. Data Tests and Computational Results**

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13 The EABCA was programmed in Microsoft Visual C# 2010 Express and  
14 executed on a 2.4 GHz Intel i7 Duo with 8 GB memory. We tested the algorithm on a  
15 set of benchmark instances developed by Gélinas et al. (1995) for VRPBTW.  
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#### 20 21 *3.1 Parameter Setting*

22 A small study on parametrization of our algorithm is carried out and shown in  
23 this section. The crucial parameters ( $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\lambda$ ) are varied and their solutions are  
24 compared using a randomly selected large problem with 10% backhauls.  
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30 **Figure 4 The relationship between the fitness value and the ratio of parameters  $\alpha$ ,**  
31  **$\beta$ , and  $\gamma$ .**  
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37 The parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  are the proportion weights for traveling time, idle  
38 time, and urgency of delivery respectively when  $\alpha + \beta + \gamma = 1$ ,  $\alpha > 0$ ,  $\beta > 0$ ,  $\gamma > 0$ .  
39 Therefore, we analyzed the ratio of these parameters instead of individual value  
40 analysis. The relationship of the fitness and some ratios of  $\alpha$ ,  $\beta$ ,  $\gamma$  parameters is shown  
41 in Figure 4. The experiment indicated that the performance of this algorithm is better  
42 when  $\alpha$  parameter is weighted more than the others, and it can produce the best solution  
43 when  $\alpha : \beta : \gamma = 0.4 : 0.3 : 0.3$ . Thus, these parameters are set as  $\alpha = 0.4$ ,  $\beta = 0.3$ ,  
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54  $\gamma = 0.3$  in this paper.  
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4 **Figure 5 The relationship between the fitness value and parameter  $\lambda$ , and**  
5 **comparison  $\lambda$ -interchange between with and without 1-move intra-route.**  
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11 The relationship between the fitness value and parameter  $\lambda$  is shown in Figure 5.  
12 The smaller  $\lambda$  is, the more difficult it is for EABCA to obtain better solution since the  
13 number of customers to be exchanged between routes is limited. Thus, the value of  
14 parameter  $\lambda = 4$  is set in this paper. Moreover, the comparison of  $\lambda$ -interchange with and  
15 without 1-move intra-route is also shown in this figure. The experiment indicated that  
16 the  $\lambda$ -interchange with 1-move intra-route can produce better solution when compared  
17 with the  $\lambda$ -interchange without 1-move intra-route. Thus, the 1-move intra-route can  
18 help improve the algorithm performance.  
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28 Both number of employed bees and the number of onlooker bees are set to be  
29 50, which is recommended by Karaboga and Basturk (2008) for good performance of  
30 ABC. For the other parameters, we set them as follows: *the limit time = 20*, and  
31 maximum number of iterations = 200.  
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### 40 3.2 Computational Results

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42 Tables 1-3 compare the results of EABCA with the original ABCA  
43 (Tuntitippawan and Asawarungsaengkul, 2016b), HMA (Küçüköğlü and Öztürk, 2015),  
44 and DEA (Küçüköğlü and Öztürk, 2014). In addition, the EABCA solutions are  
45 compared with the best-known solutions that are collected from many papers, namely  
46 Thangiah et al. (1996), Potvin et al. (1996b), Ropke and Pisinger (2006), Küçüköğlü  
47 and Öztürk (2014), Küçüköğlü and Öztürk (2015), Tuntitippawan and  
48 Asawarungsaengkul (2016b), Worawattawechai et al (2016a), and Worawattawechai et  
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4 al (2016b). The NV column represents the number of vehicles used in the solution. The  
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6 best distance of the proposed algorithm from 10 independent runs is shown in the Best  
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8 Distance column. The %Gap\_BKS column denotes the gap percentage between the  
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10 considered solution and the best-known solution. A negative number in this column  
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12 means the considered algorithm obtained a new best-known solution. Specifically, the  
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14 %Gap\_BKS is computed by the formula:  
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$$17 \quad \% \text{Gap\_BKS} = \frac{(\text{the considered solution}) - (\text{the best known solution})}{\text{the best known solution}} \times 100.$$

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24 **Table 1 Computational results of the EABCA in VRPBTW with 25 customers.**

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26 **Table 2 Computational results of the EABCA in VRPBTW with 50 customers.**

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28 **Table 3 Computational results of the EABCA in VRPBTW with 100 customers.**

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32 Tables 1-3 show the comparisons for small-, medium-, and large-sized problems  
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34 respectively. The results obtained from the comparison can be summarized as follow.

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- When compared with the original ABC, the EABCA obtained 34 equivalent or better solutions out of 34 problems presented in the ABCA paper (100%).
  - When compared with the HMA, the EABCA obtained 36 equivalent or better solutions out of 45 problems presented in the HMA paper (80%).
  - When compared with the DEA, the EABCA obtained 38 equivalent or better solutions out of 45 problems presented in the DEA paper (84%).

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52 The results show that the EABCA is superior to ABCA in terms of solution  
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54 quality, and it is competitive with the other heuristics in the literature.  
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4 To evaluate the efficiency of EABCA, the comparison between the best-known  
5 solutions in the literature and the solutions obtained from the proposed heuristic in this  
6 paper is also shown in %Gap\_BKS column of Tables 1-3. The results obtained from the  
7 comparison can be summarized as follow.  
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14 • For small problems with 25 customers in Table 1, the EABCA obtained 14  
15 solutions that were equivalent to the best-known solutions out of 15 instances. In  
16 other words, the EABCA solutions for all benchmark instances were equivalent  
17 to the best-known solutions except for only one from the R101 problem with  
18 30% backhauls.  
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- 25 • For medium problems with 50 customers in Table 2, the EABCA obtained 13  
26 solutions that were equivalent to or better than the best-known solutions out of  
27 15 instances. Moreover, the proposed algorithm could find 7 new best-known  
28 solutions. In general, the EABCA outperformed the existing algorithms in  
29 medium problem set.  
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- 36 • For large problems with 100 customers in Table 3, most of the EABCA  
37 solutions were not as good as the best-known solutions except for 6 cases where  
38 the new best-known solutions were obtained, namely the R101 problem with  
39 10% and 30% backhauls, all problems in the R102, and the R105 problem with  
40 10% backhauls.  
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48 Summarily, the EABCA outperformed the existing algorithms in terms of  
49 solution quality in many problems as it obtained 33 equivalent or new best-known  
50 solutions out of 45 instances (73.33%) while others did not perform as well (ABCA  
51 58.82%, HMA 53.33%, and DEA 13.33%). Moreover, our algorithm found the optimal  
52 solutions for some instances. The average computational time of EABCA for 25, 50,  
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4 and 100 customers are 15.16, 87.64, and 275.47 (seconds) respectively. The gap of the  
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6 total distance between the best-known solutions and the proposed solutions are within  
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8 0.5% of the best-known solutions (0.06% for 25 nodes, 0.12% for 50 nodes and 0.48%  
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10 for 100 nodes). It is computed by the formula:

$$11 \quad \%Gap_{total} = \frac{(the\ summation\ of\ all\ EABC\ solution) - (the\ summation\ of\ all\ best\ known\ solution)}{the\ summation\ of\ all\ best\ known\ solution} \times 100.$$

### 12 13 14 15 16 17 18 *3.3 Result Discussion*

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20 When comparing the results of enhanced version of ABCA with the original one  
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22 proposed by Tuntitippawan and Asawarungsaengkul (2016b), the EABCA was superior  
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24 to original ABCA in terms of solution quality. We speculated that the forbidden list  
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26 strategy in generating process, the sequential search strategy for onlooker bees, and the  
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28 intra-route and inter-route exchange combination strategy for the local search in the  
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30 EABCA indeed helped extend the exploration on the solution space to obtain the better  
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32 solutions. Note that although the sequential search of onlookers increases the chance of  
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34 finding great solutions, it also leads to larger computational time. Further study is  
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36 needed to analyze the tradeoffs and compare the computational time with the original  
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38 ABCA.  
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42 When comparing the results of EABCA with the other methods in terms of  
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44 solution quality, we found that the performance of our algorithm is better than the HMA  
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46 and DEA for small- and medium-sized problems while comparable with the HMA and  
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48 the DEA in the large-sized problems. We speculated that there are four main reasons  
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50 EABCA contributes the successful results. First, the EABCA is a population-based  
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52 heuristic which starts with a number of unduplicated initial solutions. Therefore, it can  
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54 explore more in the solution space and get more chance to obtain the better solutions.  
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4 Second, the EABCA applied the combination of intra-route and inter-route exchange as  
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6 the neighborhood search. Hence, this strategy can extend the regions of the search space  
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8 to increase the chance for finding a better solution. Third, the high-quality solutions are  
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10 used more often than the low-quality ones to produce an improved solution in the  
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12 onlooker bee stage. Thus, the regions of the search space are searched in shorter time  
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14 and in detail. Forth, the stalled solutions are removed from the population and a new  
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16 solution from random generating is added to the population in the scout bee stage. This  
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18 process provides global search ability and prevents the search from premature  
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20 convergence problem.  
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#### 26 **4. Conclusions**

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28 In this study, we present the enhanced artificial bee colony algorithm (EABCA)  
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30 to solve the VRPBTW problem. Three strategies are proposed in EABCA, which are a  
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32 forbidden list, the sequential search for onlookers, and the combination of 1-move intra-  
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34 route exchange and  $\lambda$ -interchange technique. The computational results show that  
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36 EABCA was superior to original ABCA proposed by Tuntitippawan and  
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38 Asawarungsaengkul (2016b), and it was competitive with the other heuristics in terms  
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40 of solution quality. Moreover, EABCA solutions were compared with the best-known  
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42 solutions in the literature. Results show that EABCA obtained 33 equivalent or new  
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44 best-known solutions out of 45 problems (73.33%). In general, when compared with  
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46 existing algorithms, EABCA gave better performance in medium-sized problems and  
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48 comparable performance in small-sized problem. Thus, the EABCA can be applied  
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50 effectively to small- and medium-sized problems.  
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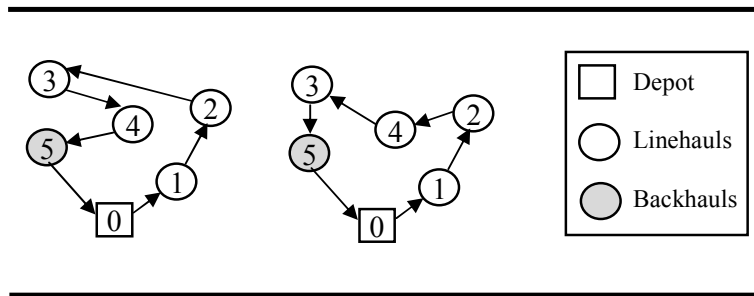


Figure 1 An example of the 1-move intra-route exchange.

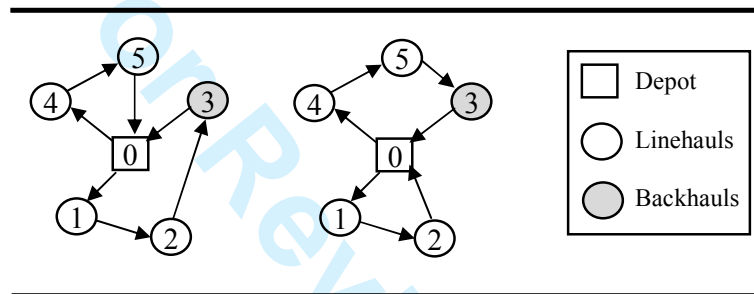


Figure 2 An example of the operation (1, 0).

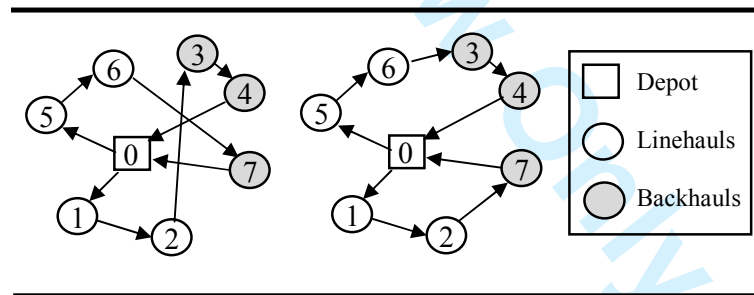


Figure 3 An example of the operation (2, 1).

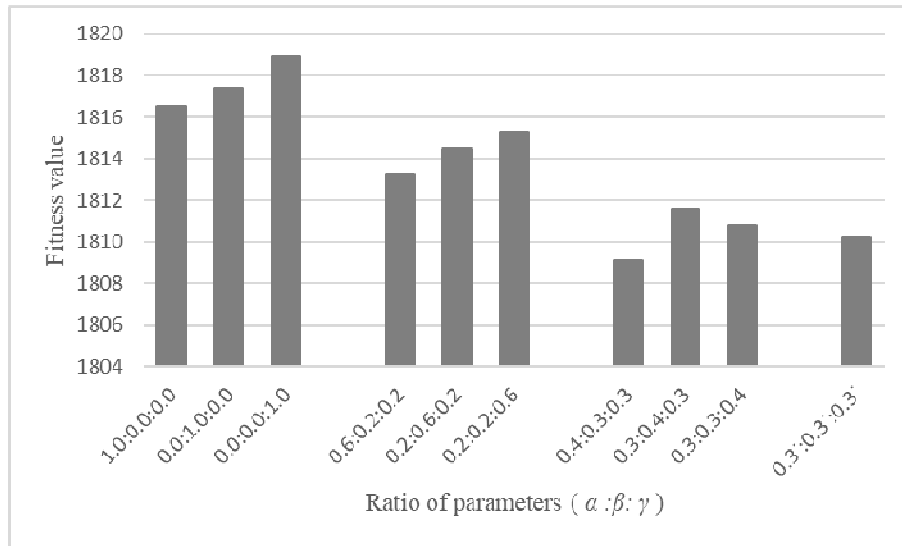


Figure 4 The relationship between the fitness value and the ratio of parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ .

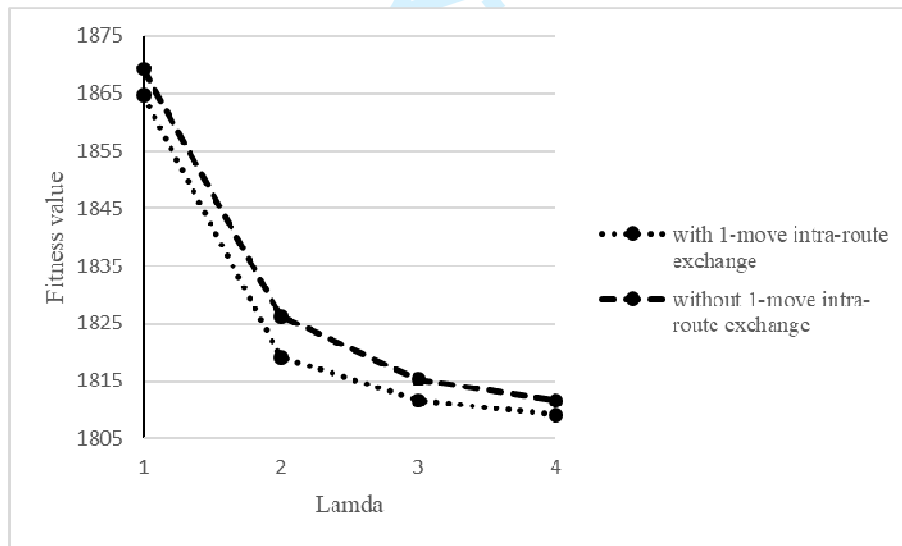


Figure 5 The relationship between the fitness value and parameter  $\lambda$ , and comparison  $\lambda$ -interchange between with and without 1-move intra-route.

| Problem | BH. (%) | Optimal Solution   | BKS                | EABCA         |    | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> | %Gap_BKS |                   |                  |                  |
|---------|---------|--------------------|--------------------|---------------|----|-------------------|------------------|------------------|----------|-------------------|------------------|------------------|
|         |         |                    | Distance           | Best Distance | NV | Distance          | Distance         | Distance         | EABCA    | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> |
| R101    | 10      | 643.4 <sup>c</sup> | 643.4 <sup>a</sup> | 643.4         | 9  | 643.4             | 643.4            | 643.4            | 0.00%    | 0.00%             | 0.00%            | 0.00%            |
|         | 30      | 711.1 <sup>c</sup> | 717.0 <sup>b</sup> | 721.8         | 10 | 721.8             | 721.8            | 721.8            | 0.67%    | 0.67%             | 0.67%            | 0.67%            |
|         | 50      | 674.5 <sup>c</sup> | 676.8 <sup>f</sup> | 676.8         | 10 | 676.8             | 676.8            | 676.8            | 0.00%    | 0.00%             | 0.00%            | 0.00%            |
| R102    | 10      | 563.5 <sup>c</sup> | 563.5 <sup>c</sup> | 563.5         | 7  | 563.5             | 563.5            | 565.3            | 0.00%    | 0.00%             | 0.00%            | 0.32%            |
|         | 30      | 622.3 <sup>c</sup> | 628.1 <sup>c</sup> | 628.1         | 9  | 628.1             | 628.1            | 629.0            | 0.00%    | 0.00%             | 0.00%            | 0.14%            |
|         | 50      | 584.4 <sup>c</sup> | 584.4 <sup>c</sup> | 584.4         | 8  | 584.4             | 584.4            | 585.3            | 0.00%    | 0.00%             | 0.00%            | 0.15%            |
| R103    | 10      | 476.6 <sup>c</sup> | 476.6 <sup>c</sup> | 476.6         | 5  | 476.6             | 478.8            | 489.0            | 0.00%    | 0.00%             | 0.46%            | 2.13%            |
|         | 30      | 507.0 <sup>c</sup> | 507.0 <sup>c</sup> | 507.0         | 7  | 507.0             | 507.0            | 510.9            | 0.00%    | 0.00%             | 0.00%            | 0.77%            |
|         | 50      | 475.6 <sup>c</sup> | 483.0 <sup>c</sup> | 483.0         | 6  | 483.0             | 483.0            | 495.0            | 0.00%    | 0.00%             | 0.00%            | 2.48%            |
| R104    | 10      | 452.5 <sup>c</sup> | 452.5 <sup>d</sup> | 452.5         | 5  | 453.8             | 453.8            | 459.1            | 0.00%    | 0.29%             | 0.29%            | 1.46%            |
|         | 30      | 467.6 <sup>c</sup> | 468.5 <sup>c</sup> | 468.5         | 6  | 468.5             | 468.5            | 469.6            | 0.00%    | 0.00%             | 0.00%            | 0.23%            |
|         | 50      | 446.8 <sup>c</sup> | 446.8 <sup>c</sup> | 446.8         | 5  | 446.8             | 446.8            | 458.7            | 0.00%    | 0.00%             | 0.00%            | 2.66%            |
| R105    | 10      | 565.1 <sup>c</sup> | 565.1 <sup>a</sup> | 565.1         | 7  | 565.1             | 565.1            | 565.1            | 0.00%    | 0.00%             | 0.00%            | 0.00%            |
|         | 30      | 623.5 <sup>c</sup> | 623.5 <sup>c</sup> | 623.5         | 8  | 628.0             | 623.5            | 630.2            | 0.00%    | 0.72%             | 0.00%            | 1.07%            |
|         | 50      | 591.1 <sup>c</sup> | 591.1 <sup>d</sup> | 591.1         | 8  | 591.1             | 592.1            | 598.5            | 0.00%    | 0.00%             | 0.17%            | 1.25%            |

<sup>a</sup> Obtained from Potvin et al. (1996b)

<sup>b</sup> Obtained from Thangiah et al. (1996)

<sup>c</sup> Obtained from Küçükoğlu and Öztürk (2015)

<sup>d</sup> Obtained from Worawattawechai et al (2016a)

<sup>e</sup> Obtained from Tuntitippawan and Asawarungsaengkul (2016)

<sup>f</sup> Obtained from Küçükoğlu and Öztürk (2014)

**Table 1 Computational results of the EABCA in VRPBTW with 25 customers.**



| Problem | BH. (%) | Optimal Solution    | BKS                 | EABCA         |    | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> | %Gap_BKS |                   |                  |                  |
|---------|---------|---------------------|---------------------|---------------|----|-------------------|------------------|------------------|----------|-------------------|------------------|------------------|
|         |         |                     | Distance            | Best Distance | NV | Distance          | Distance         | Distance         | EABCA    | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> |
| R101    | 10      | 1122.3 <sup>c</sup> | 1133.3 <sup>g</sup> | 1133.3        | 15 | 1134.0            | 1135.8           | 1138.3           | 0.00%    | 0.06%             | 0.22%            | 0.44%            |
|         | 30      | 1191.5 <sup>c</sup> | 1191.6 <sup>c</sup> | 1191.6        | 16 | 1191.6            | 1191.6           | 1245.8           | 0.00%    | 0.00%             | 0.00%            | 4.55%            |
|         | 50      | 1168.6 <sup>c</sup> | 1183.9 <sup>a</sup> | 1183.9        | 16 | 1183.9            | 1183.9           | 1183.9           | 0.00%    | 0.00%             | 0.00%            | 0.00%            |
| R102    | 10      | 974.7 <sup>c</sup>  | 976.5 <sup>e</sup>  | 976.5         | 12 | 976.5             | 976.8            | 978.7            | 0.00%    | 0.00%             | 0.03%            | 0.23%            |
|         | 30      | 1024.8 <sup>c</sup> | 1024.8 <sup>b</sup> | 1054.6        | 14 | 1054.6            | 1046.0           | 1046.0           | 2.91%    | 2.91%             | 2.07%            | 2.07%            |
|         | 50      | 1057.2 <sup>c</sup> | 1059.7 <sup>a</sup> | 1059.7        | 14 | 1059.7            | 1061.6           | 1153.0           | 0.00%    | 0.00%             | 0.18%            | 8.80%            |
| R103    | 10      | 811.4 <sup>c</sup>  | 815.5 <sup>c</sup>  | 812.3         | 9  | 821.6             | 815.5            | 831.1            | -0.39%   | 0.75%             | 0.00%            | 1.91%            |
|         | 30      | 882.8 <sup>c</sup>  | 887.1 <sup>e</sup>  | 886.2         | 11 | 887.1             | 889.3            | 895.1            | -0.10%   | 0.00%             | 0.25%            | 0.90%            |
|         | 50      | 882.1 <sup>c</sup>  | 885.1 <sup>e</sup>  | 883.0         | 11 | 885.1             | 887.7            | 887.7            | -0.24%   | 0.00%             | 0.29%            | 0.29%            |
| R104    | 10      | -                   | 687.7 <sup>c</sup>  | 685.9         | 7  | -                 | 687.7            | 688.7            | -0.26%   | -                 | 0.00%            | 0.15%            |
|         | 30      | -                   | 736.8 <sup>c</sup>  | 734.8         | 8  | -                 | 736.8            | 737.7            | -0.27%   | -                 | 0.00%            | 0.12%            |
|         | 50      | 733.6 <sup>c</sup>  | 734.5 <sup>g</sup>  | 733.6         | 8  | 739.3             | 738.2            | 742.2            | -0.12%   | 0.65%             | 0.50%            | 1.05%            |
| R105    | 10      | 970.6 <sup>c</sup>  | 972.8 <sup>f</sup>  | 976.2         | 11 | 985.2             | 978.5            | 972.8            | 0.35%    | 1.27%             | 0.59%            | 0.00%            |
|         | 30      | 1007.5 <sup>c</sup> | 1024.7 <sup>e</sup> | 1019.9        | 12 | 1024.7            | 1026.7           | 1030.0           | -0.47%   | 0.00%             | 0.20%            | 0.52%            |
|         | 50      | 993.4 <sup>c</sup>  | 993.4 <sup>e</sup>  | 993.4         | 11 | 993.4             | 996.2            | 1022.2           | 0.00%    | 0.00%             | 0.28%            | 2.90%            |

<sup>a</sup> Obtained from Potvin et al. (1996b)

<sup>b</sup> Obtained from Thangiah et al. (1996)

<sup>c</sup> Obtained from Küçüköğlü and Öztürk (2015)

<sup>d</sup> Obtained from Worawattavechai et al. (2016a)

<sup>e</sup> Obtained from Tuntitippawan and Asawarungsangkul (2016)

<sup>f</sup> Obtained from Küçüköğlü and Öztürk (2014)

<sup>g</sup> Obtained from Worawattavechai et al. (2016b)

**Table 2 Computational results of the EABCA in VRPBTW with 50 customers.**

| Problem | BH. (%) | Optimal Solution    | BKS                 |        | EABCA         |        | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> | %Gap_BKS |                   |                  |                  |
|---------|---------|---------------------|---------------------|--------|---------------|--------|-------------------|------------------|------------------|----------|-------------------|------------------|------------------|
|         |         |                     | Distance            |        | Best Distance | NV     | Distance          | Distance         | Distance         | EABCA    | ABCA <sup>e</sup> | HMA <sup>c</sup> | DEA <sup>f</sup> |
| R101    | 10      | 1767.9 <sup>c</sup> | 1811.6 <sup>f</sup> | 1809.1 | 24            | 1818.6 | 1811.6            | 1811.6           | -0.14%           | 0.39%    | 0.00%             | 0.00%            |                  |
|         | 30      | 1877.6 <sup>c</sup> | 1885.2 <sup>d</sup> | 1885.0 | 24            | 1904.5 | 1891.1            | 1925.9           | -0.01%           | 1.02%    | 0.31%             | 2.16%            |                  |
|         | 50      | 1895.1 <sup>c</sup> | 1905.9 <sup>a</sup> | 1921.2 | 25            | 1928.2 | 1911.2            | 1930.2           | 0.80%            | 1.17%    | 0.28%             | 1.27%            |                  |
| R102    | 10      | 1600.5 <sup>c</sup> | 1623.7 <sup>c</sup> | 1620.8 | 20            | 1640.7 | 1623.7            | 1649.8           | -0.18%           | 1.05%    | 0.00%             | 1.61%            |                  |
|         | 30      | 1639.2 <sup>c</sup> | 1705.6 <sup>g</sup> | 1693.4 | 21            | 1717.3 | 1724.0            | 1758.2           | -0.72%           | 0.69%    | 1.08%             | 3.08%            |                  |
|         | 50      | 1721.3 <sup>c</sup> | 1746.0 <sup>b</sup> | 1738.7 | 21            | 1752.2 | 1759.8            | 1777.1           | -0.42%           | 0.36%    | 0.79%             | 1.78%            |                  |
| R103    | 10      | -                   | 1346.9 <sup>c</sup> | 1355.2 | 17            | -      | 1346.9            | 1356.3           | 0.62%            | -        | 0.00%             | 0.70%            |                  |
|         | 30      | -                   | 1385.9 <sup>c</sup> | 1408.6 | 17            | -      | 1385.9            | 1389.2           | 1.64%            | -        | 0.00%             | 0.24%            |                  |
|         | 50      | -                   | 1456.5 <sup>h</sup> | 1463.7 | 18            | -      | 1465.0            | 1465.0           | 0.49%            | -        | 0.58%             | 0.58%            |                  |
| R104    | 10      | -                   | 1084.2 <sup>h</sup> | 1119.6 | 13            | -      | 1093.4            | 1105.4           | 3.27%            | -        | 0.85%             | 1.96%            |                  |
|         | 30      | -                   | 1136.6 <sup>c</sup> | 1148.6 | 14            | -      | 1136.6            | 1146.5           | 1.06%            | -        | 0.00%             | 0.87%            |                  |
|         | 50      | -                   | 1187.7 <sup>d</sup> | 1207.4 | 14            | -      | 1189.6            | 1199.6           | 1.66%            | -        | 0.16%             | 1.00%            |                  |
| R105    | 10      | -                   | 1516.0 <sup>c</sup> | 1514.3 | 18            | -      | 1516.0            | 1527.7           | -0.11%           | -        | 0.00%             | 0.77%            |                  |
|         | 30      | -                   | 1581.5 <sup>c</sup> | 1594.5 | 17            | -      | 1581.5            | 1582.6           | 0.82%            | -        | 0.00%             | 0.07%            |                  |
|         | 50      | -                   | 1604.1 <sup>c</sup> | 1607.2 | 18            | -      | 1604.1            | 1608.6           | 0.19%            | -        | 0.00%             | 0.28%            |                  |

<sup>a</sup> Obtained from Potvin et al. (1996b)

<sup>b</sup> Obtained from Thangiah et al. (1996)

<sup>c</sup> Obtained from Küçüköğlü and Öztürk (2015)

<sup>d</sup> Obtained from Worawattavechai et al. (2016a)

<sup>e</sup> Obtained from Tuntitippawan and Asawarungsangkul (2016)

<sup>f</sup> Obtained from Küçüköğlü and Öztürk (2014)

<sup>g</sup> Obtained from Worawattavechai et al. (2016b)

<sup>h</sup> Obtained from Ropke and Pisinger (2006)

**Table 3 Computational results of the EABCA in VRPBTW with 100 customers.**