COMPUTATIONAL STUDY ON TWO-PHASE MHD BUOYANCY DRIVEN FLOW IN AN ASYMMETRIC DIVERGING CHANNEL

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<tr>
<th>Journal:</th>
<th>Songklanakarin Journal of Science and Technology</th>
</tr>
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<td>Manuscript ID</td>
<td>SJST-2018-0248.R2</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Original Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>09-Jan-2019</td>
</tr>
<tr>
<td>Complete List of Authors:</td>
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<tr>
<td>Keyword:</td>
<td>MHD, Particulate suspension, two-phase flow, diverging channel.</td>
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COMPUTATIONAL STUDY ON TWO-PHASE MHD BUOYANCY DRIVEN FLOW IN AN ASYMMETRIC DIVERGING CHANNEL

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ABSTRACT

In this paper the problem of a two- dimensional steady viscous, incompressible two-phase flow of a particulate suspension in an asymmetric diverging channel with a heat source is considered. The differential equations governing the flow are non-dimentionalized by employing suitable transformations and resulting equations are solved numerically, using Runge-Kutta Shooting technique. The influence of Magnetic parameter, Reynolds number, Cross flow Reynolds number, Grashof number, heat source parameter, Prandtl number are exhibited graphically and discussed for velocity and temperature profiles for both fluid as well as particle phases. Computational values for skin friction coefficient, Nusselt number are obtained and presented in tabular form and discussed. This study plays an important role in many engineering and biological fields such as cooling of nuclear reactors, chemical and food industries, blood flow through capillaries and arteries.

KEYWORDS: MHD, Particulate suspension, two-phase flow, diverging channel.
NOMENCLATURE

\( r, \theta \) Polar coordinates,

\( \alpha \) Angle of the channel,

\( \nu \) Kinematic viscosity \( (m^2 s^{-1}) \),

\( \mu \) Coefficient of viscosity \( (kg m^{-1}s^{-1}) \)

\( \rho \) Density of the fluid \( (kg m^{-3}) \),

\( u \) Fluid phase velocity \( (ms^{-1}) \)

\( u_p \) Particle phase velocity \( (ms^{-1}) \),

\( S \) Drag coefficient of the interaction for the force exerted by one face on the other

\( T \) Fluid phase temperature (K)

\( T_p \) Particle phase temperature (K)

\( U_0 \) Radial velocity along center line \( (LT^{-1}) \).

\( V_0 \) Suction/Injection velocity at \( r = r_0 (LT^{-1}) \)

\( \beta^* \) Coefficient of thermal expansion

\( \sigma \) Electric conductivity of the fluid \( (Sm^{-1}) \)

\( H_0 \) Magnetic field intensity

\( \mu_e \) Magnetic permeability of the fluid

\( \rho_p \) Density of the particle \( (Kg m^{-3}) \)

\( C_p \) Specific heat of the fluid \( (JKg^{-1} K) \)

\( C_m \) Specific heat of the particles \( (JKg^{-1} K) \)

\( Q_0 \) Heat generation coefficient, \( (w m^{-3}) \)
K  Thermal conductivity of the fluid (W m\(^{-1}\) k\(^{-1}\))

Re  Reynolds number \(\frac{U_0 r_0}{v}\)

R  Cross flow Reynolds number \(\frac{V_0 r_0}{v}\)

L  Ratio of the densities of the particle and fluid phase \(\frac{\rho_p}{\rho}\)

\(\beta\)  Fluid particle interaction parameter for velocity \(\frac{sr^2}{\nu}\)

\(M^2\)  Magnetic parameter \(\frac{\sigma H_0^2 \mu_e^2 r^2}{\rho \nu}\)

\(Gr\)  Grashof number \(\frac{g \beta' T w r^3}{\nu^2}\)

Pr  Prandtl number \(\frac{\mu c_p}{k}\)

\(Q\)  Heat source parameter \(\frac{Q_0}{\rho c_p \nu}\)

\(\gamma\)  Specific heat ratio \(\frac{c_m}{c_p}\)

INTRODUCTION

Study of flow through spatially varying geometries like converging or diverging channels is of great importance in aerospace, industrial, environmental and biomechanical engineering as well as in understanding flow over rivers and canals. An incompressible viscous fluid flow between two non-parallel plates was first studied by Jeffery (1915). Banks, Drazin, and Zaturska (1988)


Two-phase flow of particulate suspension applications abound in many areas of technology: food industries, powder technology, in waste water treatment, combustion and corrosive particles in engine oil flow etc. So it is important to study fluid-particle hydromagnetic convective flows in order to study the influence of the different phases on heat transfer processes. Recently, a remarkable number of researches, Sivakumar, Sreenath, and Pushpavanam (2010), Hatami, Hosseinzadeh, Domairry, and Behnamfar (2014) have been investigated two-phase particulate flows with and without magnetic field and heat transfer analytically and numerically. Chamkha (1995) studied hydromagnetic two-phase flow in a channel. Mansour and Chamkha (2003) developed a continuum model to analyze heat generation effects on two-phase particulate suspension MHD flow through a channel. Usha, Senthilkumar and Tulapurkara (2006) investigated particulate suspension flow in a travelling wavy channel. Heat generation effects on hydromagnetic flow of a particulate suspension through isothermal-isoflux channels investigated by Chamkha and Rashidi (2010). Rawat, Bhargava, Kapoor, Beg, Beg, and Bansal (2014) presented a numerical model for steady two dimensional two-phase hydromagnetic flows and heat transfer in a particulate-suspension through a non-Darcian porous channel. Sadia, Naheed and Anwar (2017) studied compressible dusty gas along a vertical wavy surface. Krupalakshmi,

With the available literature and to best of authors knowledge, no one has studied on convective two-phase flow in an asymmetric divergent channel. Keeping in view of the above facts, mathematical model has been developed to study MHD convective two-phase particulate suspension flow in divergent channel with heat source.

**MODEL OF THE PROBLEM**

Consider steady, viscous, two-dimensional incompressible laminar two-phase flow of particulate suspension in an asymmetric diverging channel. Walls of the channel are placed at $\theta = \pm \alpha$ as shown in fig.1. Suction/injection velocities are assumed at different walls and these velocities are to be vary inversely proportional to the distance along the wall from origin of the channel. The continuity equation, the Navier–Stokes equations and the energy equation governing the flow in polar coordinates $(r, \theta)$ are: Terril (1965) and Baris (2003)
For Fluid phase

\[
\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial \theta} v = 0, \quad (1)
\]

\[
\left( \frac{\partial u}{\partial r} + \frac{\partial}{r \partial \theta} \frac{u^2}{r} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \left( \frac{\partial^2 u}{\partial r^2} + \frac{2 \partial u}{r \partial r} \right) - \frac{u}{r^2} + \frac{2 \partial v}{r \partial \theta} \right] + \frac{\rho_p S}{\rho} (u_p - u) - \frac{\sigma H_s \mu^2 u}{\rho} - g \beta T, \quad (2)
\]

\[
\left( \frac{\partial v}{\partial r} + \frac{\partial}{r \partial \theta} \frac{uv}{r} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \left( \frac{\partial^2 v}{\partial r^2} + \frac{2 \partial v}{r \partial r} \right) \right] - \frac{2 \partial u}{r^2} \frac{\partial \theta} - \frac{\rho_p S}{\rho} (v_p - v), \quad (3)
\]

\[
u \frac{\partial T}{\partial r} + \frac{\partial T}{r \partial \theta} = \frac{k}{\rho c_p} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right) + \frac{Q_0}{\rho c_p} T + \frac{\rho_p C_m}{\tau_f \rho c_p} (T_p - T), \quad (4)
\]

For Particle phase

\[
\frac{\partial}{\partial r}(r u_p) + \frac{\partial}{\partial \theta} v_p = 0, \quad (5)
\]

\[
\left( \frac{\partial u_p}{\partial r} + \frac{\partial}{r \partial \theta} \frac{u_p v_p}{r} - \frac{v_p^2}{r} \right) = -\frac{1}{\rho_p} \frac{\partial p}{\partial r} + S (u_p - u) - g \beta T_p, \quad (6)
\]

\[
\left( \frac{\partial v_p}{\partial r} + \frac{\partial}{r \partial \theta} \frac{u_p v_p}{r} + \frac{u_p v_p}{r} \right) = -\frac{1}{r \rho_p} \frac{\partial p}{\partial \theta} + S (v - v_p), \quad (7)
\]

\[
u_p \frac{\partial T_p}{\partial r} + \frac{\partial T_p}{r \partial \theta} = \frac{1}{\tau_f} (T - T_p), \quad (8)
\]

The associated boundary conditions are

\[
u = 0, u_p = 0 \quad \text{at} \ \theta = \pm \alpha \quad u(\theta = 0) = U_0
\]

\[
u = T, T_p = T_{w_p} \quad \text{at} \ \theta = \pm \alpha \quad (9)
\]

Introducing the following dimensionless variables

\[
u = \frac{U_0 r_0 f(\theta)}{r}, u_p = \frac{U_0 r_0 g(\theta)}{r}, v_p = \frac{V_p r_0}{r}, v = \frac{V_p r_0}{r}, h = \frac{T}{T_w}, H = \frac{T}{T_{w_p}}
\]

eqns. (1) – (9) are reduced into
\[ f'''' + 2 \text{Re} f'' - Rf'' + L\beta (g' - f') + (4 - M^2)f' - \frac{Gr}{\text{Re}} h' = 0, \]  
(10)

\[ g'' - 2 \frac{\text{Re}}{R} gg' - \frac{\beta}{R} (f' - g' - \text{Gr} \frac{H'}{\text{Re} R} = 0, \]  
(11)

\[ h'' - R Pr h' + Pr Qh + L\beta,\gamma Pr (H - h) = 0, \]  
(12)

\[ H' - K (h - H) = 0, \]  
(13)

Associated boundary conditions are

\[ f (\pm \alpha) = 0, f (0) = 1 \]
\[ g (\pm \alpha) = 0, h (\pm \alpha) = 1, H (\pm \alpha) = 1 \]

(14)

**SKIN FRICTION COEFFICIENT AND NUSSLETT NUMBER**

The main aim of the physical interest of the problem is analyzing drag coefficient and rate of heat transfer over surface of the channel, which are defined by skin friction \( C_f = \frac{\tau_s}{\rho U_o^2} = \frac{\mu}{\rho U_o^2} \frac{\partial u}{\partial \theta} \bigg|_{\theta = \pm \alpha} \)

and Nusselt number \( Nu = \frac{r q_s}{\kappa T_w} = \frac{\left( - \kappa \partial T \right)}{\kappa T_w} \bigg|_{\theta = \pm \alpha} \)

In non-dimensional form

\[ C_f = \frac{1}{\text{Re}} f' (\pm \alpha) \text{ and } Nu = -h' (\pm \alpha) \]

**SOLUTION METHODOLOGY**

A set of equations (10) – (13) with boundary conditions (14) are first rewrite in a system of first order equations by assuming \( f' = f (1), f'' = f (2), f''' = f (3), g = f (4), g' = f (5), h = f (6), \)
\[ h' = f(7), H = f(8), \quad H' = f(9), \, i.e. \]

\[
\begin{bmatrix}
  f(1) \\
  f(2) \\
  f(3) \\
  f(4) \\
  f(5) \\
  f(6) \\
  f(7) \\
  f(8) \\
  f(9)
\end{bmatrix}
\begin{align*}
  f' &= f(2) \\
  f'' &= -2 \text{Re} f(1) f(2) + R f(3) - L \beta (f(5) - f(2)) - (4 - M^2) f(2) + \frac{Gr}{Re} f(7) \\
  g' &= 2 \frac{\text{Re}}{R} f(4) f(5) + \frac{\beta}{R} (f(2) - f(5)) + \frac{Gr}{ReR} f(9) \\
  h' &= R \text{Pr} f(7) - \text{Pr} Q f(6) - L \beta \text{Pr} \gamma (f(8) - f(6)) \\
  f(9) \\
  H' &= K (f(7) - f(9))
\end{align*}
\]  

(15)

We choose some initial conditions \( f'(-\alpha) = c_1, f''(-\alpha) = c_2, g'(-\alpha) = c_3, \) \( h'(-\alpha) = c_4, \) and \( H'(-\alpha) = c_5 \) which are not given at initial point and integrate (15) using Runge-Kutta fourth order technique (Mallikarjuna, Rashad, Hussein, & Hariprasad Raju, 2016; Mallikarjun, Rashad, Chamkha, & Hariprasad Raju, 2016; Gholinia, Gholinia S, Hosseinzadeh, & Ganji, 2018, Ghadikolaei, Hosseinzadeh, & Ganji, 2018). The obtained results are compared at \( \alpha \) and got more difference due to wrong assumption of the initial conditions. To overcome we applied Newton Raphson method to choose the initial conditions and integrate eqns. (15) using RK4 with step size 0.001 with \( 10^{-4} \) accuracy for the solutions. To validate present code, the present results are compared in the absence of energy equation, thermal buoyancy and heat source with existing results produced by Ramprasad, subba Bhatta, Mallikarjuna, and Srinivasacharya (2017) obtained results found to be good agreement as shown in Table-1. In this section we studied the role of non-dimensional flow parameters embedded in the flow model on fluid and particle phase velocities and temperatures.
RESULTS AND DISCUSSION

For numerical calculations we fixed the non dimensional parameter values as $R=1, Re=0.5, M=1.5, Gr=5, L=1, Pr=0.71, Q=0.5, \alpha = \frac{\pi}{4}, \gamma = 0.5, \beta = 1, \beta_i = 0.5$. These values reserved as constant in whole study excluding dissimilarities in the particular figures.

Figures 2-4 depict the profiles of fluid velocity with variations in different governing parameters $M$, $R$, and $Gr$. An increase in $M$, the Lorentz force which opposes the flow increases as a result it is noted that the velocity of the fluid phase decreases as shown in fig. 2. This is in good agreement with Mir Asadullah, Umar, Naveed, Raheela, and Syed (2013), Chamkha and Rashidi (2010). It can be concluded that the flow can be controlled by imposing higher magnetic field on the boundaries. Figure 3 illustrates that an increase in $R$ the velocity of the fluid decreases in the left half of the channel where as it increases in right half of the channel enormously. Near the wall $\alpha = \frac{\pi}{4}$ the viscosity effects are very small therefore the velocity attains maximum near that wall. This is in good agreement with Terril (1965), Roy and Nayak (1982). Fig 4 elucidates that an increase in $Gr$ leads to increase gravitational force and dominates thermal buoyancy force. It causes to decelerate the fluid velocity throughout the channel.

Figures 5 to 7 represent the variation of particle phase velocity with different variations in governing parameters $M$, $R$, and $Gr$. From figure 5 it is evident that a hike in $M$, an enhancement of the particle phase velocity is observed in the left part of channel and opposite behavior is observed in right part of the channel. This is in good agreement with Mansour, and Chamkha, (2003). From figure 6 it is observed that an increase in $R$ the particle phase velocity increases in left half of the channel and it is observed reverse behavior in right half of the channel. Figure 7 displays that an increase in $Gr$, the thermal buoyancy effect increases. This gives rise to accelerate
particle phase velocity in the entire channel. The same observation has been reported in Chamkha and Rashidi (2010).

The influence of Prandtl number Pr, on fluid temperature is depicted in fig 8. It can be viewed from this figure that the temperature of fluid increases rapidly with an increment in Pr. This indicates that momentum diffusivity dominating over the fluid temperature. If Pr=0.6 the fluid is oxygen and if Pr=0.71 it is air. If Pr = 1.3 the fluid is gaseous ammonia. Fig. 9 anticipates the behavior of heat source parameter Q on fluid phase temperature. As Q increases the fluid phase temperature increases and maximum temperature is attained in the mid region of the channel. Figure 10 explains the behavior of R on fluid temperature. As R increases the fluid temperature gradually decreases in entire channel. From figure 11 it is observed that an increase in Pr the temperature of the particle phase get increased over the left part of channel and decreases in the right part of the channel. This means momentum diffusivity more in right part of the canal and less in left part of the channel. An increment in Q enhances the particle temperature in left side of the channel and opposite trend is observed in the counter part of the channel as demonstrated in figure 12. Figure 13 demonstrates that an increase in R the particle phase temperature decreases in the left half of the channel and increases right half of the channel.

From Table-2 it is observed that an increase in R, the skin friction coefficient (Cf) decreases and enhances the Nusselt number (Nu) near both the walls. As Re increases skin friction coefficient decreases on left wall and increases on right wall and no change in Nusselt number over both the walls. As $\beta$ increases Cf decreases near both the walls, but Nu remains constant at both the walls. It indicates that interaction of fluid and particles does not influence rate of heat transfer over the walls. An increment in $\beta$, skin friction coefficient get increased on the left wall and get decreased on right wall but decrease in Nusselt number at both the walls.
From Table-3 it is noted that as $\gamma$ increases skin friction coefficient increases on left wall and decreases on right wall. Same behavior observed on Nusselt number. An increase in $L$ skin friction coefficient increases on left wall and decreases on right wall. The same results are observed on Nusselt number. An increment in $Q$ depreciates the skin friction coefficient and Nusselt number at left wall whereas reverse behavior is observed at right wall. As $M$ increases skin friction coefficient decreases on both the walls. Nusselt number values do not change at both the walls for increasing values of $M$.

CONCLUSIONS

In this paper, the flow of a viscous incompressible fluid through a divergent channel in a particulate suspension with MHD and heat generation has been discussed. Numerical method has been applied to solve non-linear differential equations by non-dimensionalising using suitable transformations.

The conclusions of present study as follows.

- An increase in $Gr$, the fluid velocity decreases whereas the particle phase velocity increases.
- An increase in $M$ increases fluid as well as particle phase velocities.
- An increment in $M$ decelerates fluid velocity. But particle velocity inclines in left part of the channel declines in the right part of the channel.
- An increase in Cross flow Reynolds number the fluid velocity decreases near left boundary and increases near right boundary. Similar behavior is noted in the case of particle temperature. As $R$ increases a decline in fluid temperature is observed.
- An increase in $R$, skin friction and $Nu$ increases on both the walls. An increase in $M$ the reverse behavior is observed on both walls. An increase in $\beta, \gamma, L$ skin friction and $Nu$ increases on left wall and decreases on right wall.
REFERENCES


Terril, R. M. (1965). Slow Laminar Flow in a Converging or Diverging Channel with Suction at One Wall and Blowing at the Other Wall. *ZAMP*, 16(2), 306-308. doi:10.1007/BF01587656


### TABLE 1: Comparison results of skin friction coefficient for $M=0$, $Gr=0$, $Pr=0$, $Q=0$, $\gamma = 0$, $\beta = 1$

<table>
<thead>
<tr>
<th>R</th>
<th>Re</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>L</th>
<th>Present Results</th>
<th>Ramprasad, SubbaBhatta, Mallikarjuna, Srinivasacharya (2017)</th>
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<td>$f'(-\alpha)$</td>
<td>$f'(\alpha)$</td>
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Table-2: Skin friction and Nusselt number values for different values of $R$, $Re$, $\beta$ and $\beta_i$.

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**Table-3:** Skin friction and Nusselt number values for different values of $\gamma$, L, Q and M

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**Fig.1** Geometry of the flow

**Fig.2** Effect of M on fluid phase velocity
Fig. 3 Effect of R on fluid phase velocity

Fig. 4 Effect of Gr on fluid phase velocity
Fig. 5 Effect of $M$ on particle phase velocity

Fig. 6 Effect of $R$ on particle phase velocity;
**Fig. 7** Effect of Gr on particle phase velocity

**Fig. 8** Effect of Pr on fluid phase temperature
Fig.9 Effect of Q on fluid phase temperature

Fig.10 Effect of R on fluid phase temperature
**Fig. 11** Effect of Pr on particle phase temperature

**Fig. 12** Effect of Q on particle phase temperature
Fig. 13 Effect of R on particle phase temperature