### ANALYTICAL METHOD FOR CALCULATING SCALLOP HEIGHT OF HELICAL TOROIDAL CUTTER IN FIVE-AXIS MILLING

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ANALYTICAL METHOD FOR CALCULATING SCALLOP HEIGHT
OF HELICAL TOROIDAL CUTTER IN FIVE-AXIS MILLING

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ABSTRACT

This paper proposed a method to determine the path scallop of toroidal cutter in five-axis milling during a complex surface machining. Mathematics algorithm was developed by taking into consideration the impact of helical angle and inclination angle. The applicability of the proposed method was tested using two model parts with diverse surface profiles. The result showed that the method was effectively used to generate the path scallop data. Moreover, the program simulation could also generate the shape of machined surface. The effect of helical angle to the scallop height was also tested and the result showed that the helical angle tends to decrease the scallop height. On the other hand, larger inclination angle produced larger scallop height. The verification test using Siemens-NX proved that the method was accurate.

Keywords: scallop height, toroidal cutter, grazing toroidal approximation, five-axis milling

INTRODUCTION

Generally, there are several variables mainly used to indicate the quality of the machined surface, such as: 1) machining tolerance, 2) scallop height, and 3) surface
roughness. In five-axis machining, the scallop height is the most substantial variable in defining the quality of machined surface. It is affected by: 1) the geometry of cutting tool, 2) the tool orientation, 3) the geometry of part surface, and 4) the length between two successive tool path (step over) (Hendriko, 2017a.) Therefore, the scallop must be controlled so that the expected surface quality can be obtained. The scallop height is difficult to calculate in five-axis milling due to the complicated part surface and tool orientation. Hence, an accurate method to calculate the scallop height is still a challenge in achieving optimal tool path during sculptured surface machining.

In modeling and computing the scallop height, accurate cutter workpiece engagement (CWE) information is very important. Precise geometric information has a direct influence on the precision of scallop height calculations. There are three common methods used for calculating CWE: solid model, discrete method, and analytical approach. Erdim and Sullivan (2012, 2013) used solid modeler based composite adaptive sampled distance fields to calculate the geometric modeling in five-axis milling. Meanwhile Aras and Albedah (2014) proposed closed boundary representation to calculate the surface intersection between workpiece material and cutting tool. Other researcher have used discrete methods such as Z-mapping and modified Z-mapping. Kim, Cho, and Chu (2000) used a Z-mapping method to determine the shape of CWE. Then, Wei, Wang, and Cai (2013) proposed modified Z-mapping algorithm to define CWE in sculptured surfaces. Solid model produces accurate result, but it also has the drawback on expensive computational cost. Therefore, many researchers used discrete methods because it is computationally more efficient than solid model. However, the computational time of discrete method increases intensly if the precision and accuracy are refined.
Considering these problems, some researchers were attracted to develop analytical approach in calculating the scallop height and cut geometry in five-axis milling. As compared to discrete approaches and solid model, analytical approach was much faster and more accurate. Several studies (Hendriko, 2013, 2015, 2017a, 2017b; Hendriko, Kiswanto, & Duc, 2017; Kiswanto, Hendriko, & Duc, 2014a, 2014b, 2015) developed analytical method, which was called the analytical boundary method (ABS), to calculate the cut geometry of flat-end cutter and toroidal cutter during sculptured surface. From a series of tests, it was found that the proposed method is applicable to calculate CWE accurately. Furthermore, the computational cost of ABS was proven much cheaper than Z-mapping method.

Many studies have been performed to calculate the scallop height. Several researchers (Hricora & Napstkova, 2015; Bedi, Ismail, & Mahjoob, 1997) investigated the effectiveness of inclined flat-end cutter in the milling of free-form part surfaces. The results showed that inclined flat-end cutter generates smaller scallops as compared to ball-end cutter. Other studies (Wang, Zhang, & Yan, 2016; Yigit & Lazoglu, 2015) proposed method to define the scallop height of ball-end cutter for achieving optimal step-over. Tunc and Budak (2009) investigated the effect of cutter posture angle to the scallop height in five-axis milling. The result showed that the cutter posture angle influences the scallop height significantly. In contrast to the cutter posture angle, the study to investigate the effect of helical angle to the scallop height is still very lack.

Most of studies in analytical approach addressed the issue of scallop height for toroidal cutter by simply assumed that the curvature was constant and cutter geometry was approximated by two common primitive geometries, either circle or ellipse. Senatore, Segonds, Rubio, and Dessein (2012) calculated the effective radius of toroidal
cutter due to the inclination angle for representing the swept curve. Then, the scallop height with respect to the radius of part surface was determined using an approximated swept curve, and finally, an optimal step-over can be calculated. Others studies (Hricora & Napstkova, 2015; Bedi, Ismail, Mahjoob, & Chen, 1997; Wang et al., 2016; Yigit & Lazoglu, 2015; Ozturk et al., 2009; Senatore et al., 2012; Chiou & Lee, 2002; Weinert, Du, Damm, & Stautner, 2004) used ellipse to represent the inclined flat-end and ball-end mills. Many studies proved that parametric equation of ellipse curve could be used precisely to represent the swept curves of inclined flat-end and ball-end mill. However, this approach is not applicable for toroidal cutter. Geometrically toroidal cutter is more complex than flat-end mill and ball-end mill because it is constructed by two faces, cylindrical face and toroidal face. Consequently, calculating the swept curve when the cutter was set with an inclination angle becomes much more complicated. This issue will be proven in the section of Implementation and discussion.

In this study, the swept curve was defined by extending the method to identify the lower engagement point (LE-point), which is called the grazing method, as proposed by Kiswanto et al. (2014a). The scallop height was defined as the length between the intersection point of cutting path to the surface normal. Therefore, the algorithm of grazing method was then extended so that it could be used to determine the intersection point between two consecutive tool path. The intersection point of cutting path was determined using a combination of coordinate transformation system and algorithm of swept curve. Meanwhile the surface normal at instantaneous tool position was determined based on the instantaneous surface shape. In this study, the shape of part surface at instantaneous cutter position was defined using three normal vectors mathematically. The analytical boundary simulation (ABS), as proposed by Kiswanto et al. (2015) was used
for defining the shape of part surface and calculating the normal distance. In this study, the algorithm to determine the scallop height was developed by taking into consideration the effect of tool inclination angle and helical angle.

SWEPT CURVE CALCULATION

In this study, the algorithm was derived for toroidal cutter. Typically, the surface of toroidal-end cutter was constructed by cylindrical and toroidal faces as depicted in Figure 1a. However, despite it is constructed by two faces, the scallop height was only created by the engagement between the toroidal side and the workpiece material. The shape of toroidal face in the cutter coordinate frame (CCF) was defined using the following equation,

\[
G_T(\varphi; \lambda) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (r_m + r \sin \lambda) \sin \varphi \\ (r_m + r \sin \lambda) \cos \varphi \\ r - r \cos \lambda \end{bmatrix}
\]

where \( r \) is corner radius of cutter and \( r_m \) is the length between point \( T \) to point \( e \) as shown in Figure 1a. Meanwhile \( \lambda \) and \( \varphi \) denotes the toroidal angle and rotational angle, respectively.

In five-axis milling, cutter can move freely in space because it can be rotated in \( x \)-axis and \( y \)-axis. A complex part surface can be machined effectively by managing the motion of cutter with respect to the part surface normal (curvatures).

In this study, three coordinate systems were used to define the location and posture of the cutting tool as illustrated in Figure 1b. They are global coordinate frame (GCF), which is used as reference coordinate system, cutter coordinate frame (CCF), and local coordinate frame (LCF). GCF is a permanent frame which is denoted by the basis vector \( x \), \( y \), \( z \), while CCF and LCF are represented by \( u \), \( v \), \( w \) and \( X \), \( Y \), \( Z \).
respectively. The tool inclination angle ($\alpha$) is normally used in five-axis milling when machining a free-form part surface. It is the tool rotation angle formed with respect to CCF and LCF as illustrated in Figure 1c. For the purpose of coordinate system transformation from CCF to LCF, an appropriate mapping operator $[M]$ was required. The transformation operator involve the tool orientation about $x$-axis ($\theta_A$), $y$-axis ($\theta_B$) and also the tool displacement at $T$, was defined as follow,

$$[M] = \begin{bmatrix} \cos \theta_B & 0 & \sin \theta_B & x_T \\ \sin \theta_A \sin \theta_B \cos \theta_A & -\sin \theta_A \cos \theta_B & y_T \\ \cos \theta_A \sin \theta_B \sin \theta_A & \cos \theta_A \cos \theta_B & z_T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (2)

The cutter coordinate frame with orthogonal basis vector $u, v, w$, which is located at the bottom centre of cutting tool (point $T$), was determined by,

$$w = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} [0 \ 0 \ 1]^T = [\sin \alpha \ 0 \ \cos \alpha]^T$$ (3)

$$v = \frac{w \times V_T}{|w \times V_T|} ; \ u = v \times w$$ (4)

$V_T$ is the linear velocity from cutter location (CL) point to the next and it was calculated as follow,

$$V_T = \frac{CL_{(i+1)} - CL_{(i)}}{f} ; (i = 1, 2, 3, \ldots)$$ (5)

There are three points that constructing the swept envelope, they are: egress point, grazing point and ingress point (Chio & Lee, 2002; Weinert et al., 2004). In this study the algorithm to develop the swept curve was derived by improving the method proposed by Kiswanto et al. (2014a). The grazing method for determining the lower engagement point in
cut geometry calculation was extended so that it could be used to calculate the swept curve. The swept curve was constructed by a set of swept points at every rotation angle. The swept point can be defined after the toroidal angle ($\lambda$) was determined. The swept point was determined using the tangency function as follow,

$$F_{(\theta, \phi, p)} = N_{S_T(\theta, \phi, p)} \cdot V_{S_T(\theta, \phi, p)} = 0$$

(6)

where $N_{S_T(\phi)}$ is the surface normal of the cutting tool and $V_{S_T}$ is the moving vector of the cutting tool. Then, all the points at every rotation angle that constructing the swept curve were calculated using the same method. The surface normal of an arbitrary point on the cutting tool in CCF was expressed by,

$$N_{S_T} = \frac{\partial S_T/\partial \lambda}{|\partial S_T/\partial \lambda|} \times \frac{\partial S_T/\partial \varphi}{|\partial S_T/\partial \varphi|} = \begin{bmatrix} \sin \lambda \cdot \sin \varphi \\ \sin \lambda \cdot \cos \varphi \\ -\cos \lambda \end{bmatrix}$$

(7)

When Eq. (7) was mapped to the moving system, it yields to,

$$N_{S_T'(\theta, \phi, p)} = \cos(\phi) \cdot \sin \lambda \cdot v - \cos \lambda \cdot w + \sin \lambda \cdot \sin(\phi) \cdot u$$

(8)

The moving vector of an arbitrary point on the cutting tool was calculated as follow,

$$V_{S_T} = V_T + \overrightarrow{QT} \times \omega$$

(9)

where $\omega$ and $\overrightarrow{QT}$ denoted the angular velocity and the vector from $Q$ to $T$, respectively. Because the model was developed by assuming that the tool is static, hence, there was no angular motion ($\omega = 0$). Therefore, the linear velocity was equal to $f$ ($V_T = f$) and the tangency function yield to,
\[ F_{(\theta,\varphi,p)} = \sin \lambda \sin(\varphi) \cdot (V_T \cdot u) + \sin \lambda \cos(\varphi) \cdot (V_T \cdot v) + \cos \lambda \cdot (V_T \cdot w) \]

\[ = 0 \quad (10) \]

Because \( V_T \) is perpendicular to \( v \), then \( V_T \cdot v = 0 \). Then, the toroidal angle of swept curve point with respect to the rotation angle was defined as follow,

\[ \lambda (\varphi) = \tan^{-1} \left[ \frac{V_T \cdot w}{\sin(\varphi) \cdot (V_T \cdot u)} \right] \quad (11) \]

After \( \lambda (\varphi) \) was determined, then, the coordinat of swept curve point with respect to rotation angle in the GCF was calculated as follow,

\[ I_{(\varphi)}(x_I, y_I, z_I) = [M] \cdot S_T \left( \varphi_I; \lambda(\varphi) \right) \quad (12) \]

**EFFECT OF HELICAL ANGLE TO THE TOOL ORIENTATION AND SWEPT SURFACE**

The helical angle on the cutting teeth was aimed to solve the drawback of straight cutting teeth in which the cutter tooth cut the material with very strong effort since beginning. It may cause vibration due to the shock effect and a discontinuities of the load that give adverse effect to the quality of machined surface. Considering this benefit, the helical angle was widely used to overcome such problem. In solid cutting tool, the helical angle (\( \chi \)) was also called lag angle. It makes the cutter tooth cut the material gradually. The application of helical angle changes the geometry of cut. In this section, the effect of helical angle to the orientation of cutting tool and the shape of swept surface will be discussed.

**Identifying the tool orientation angles**
When the tool without helical angle, the cutting tool and the cutter tooth have the same orientation. But when the helical angle is introduced to the cutter tooth, then it changes the actual orientation of the cutter tooth. The effect of helical angle was illustrated using Figure 2a. By assuming that the feedrate is in the \(x\)-direction, then helical angle changes the orientation of cutter tooth by rotating the cutter tooth about the \(y\)-axis. The orientation of cutter tooth at every rotation angle was defined by determining the coordinate of two representatives points, \(s_X(x_{sX}, y_{sX}, z_{sX})\) and \(c(x_c, y_c, z_c)\). The representative points were located on the cutter tooth that was determined using parametric equation of cylindrical surface. Point \(s_X\) was obtained by rotating point \(s\) about the \(Y\)-axis by helical angle. Point \(s\) is located arbitrarily on the cutter tooth at when \(\varphi = 0\). Meanwhile point \(c\) is a representative point that was set at the bottom of the cutter tooth. The coordinate of point \(s\) and point \(c\) in CCF was defined as follow:

\[
s = (0, r_m, r + 5) \quad \text{and} \quad c = (0, r_m, r)
\]  

Then point \(s_X\) and point \(c\) as a function of rotation angle were calculated by transforming point \(s\) and point \(c\) about \(Y\)-axis by helical angle and about \(Z\)-axis by rotation angle. They were calculated as follows,

\[
s_X = \begin{bmatrix} x_{sX} \\ y_{sX} \\ z_{sX} \end{bmatrix} = \text{Rot}(Z, \varphi) \times \text{Rot}(Y, \chi) \times [s]
\]  

\[
s_X = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos \chi & 0 & \sin \chi \\ 0 & 1 & 0 \\ \sin \chi & 0 & \cos \chi \end{bmatrix} \times \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}
\]
The orientation of cutting tool relative to GCF due to the helical angle was determined by calculating the orientation of cutter tooth relative to the \( X\)-axis and the \( Y\)-axis as depicted in Figure 2b. They were calculated as follows,

\[
\begin{align*}
\theta_{Ax} &= \theta_x = \tan^{-1} \left( \frac{y_{sx} - y_c}{z_{sx}} \right) \\
\theta_y &= \tan^{-1} \left( \frac{x_{sx} - x_c}{z_{sx}} \right) \\
\theta_{Bx} &= \tan^{-1} \left( \frac{x_{sx} - x_c \cos \theta_{Ax}}{z_{sx}} \right) = \tan^{-1} \left( \tan \theta_y \cos \theta_{Ax} \right)
\end{align*}
\] (17) (18) (19)

In five-axis milling, the orientation of cutter can be set to any direction due to the complexity of the part surface. Two additional degree of freedom make the tool in five-axis milling can be rotated about \( X\)-axis and the \( Y\)-axis. The rotation angles about the \( X\)-axis and the \( Y\)-axis were denoted by \( \theta_A \) and \( \theta_B \), respectively. These angles were used to set up the tool to the desired orientation. However, when a helical angle was applied to the cutter tooth, then the orientation of the cutter tooth changes significantly. Therefore, the actual orientation of cutter tooth with respect to GCF need to be determined. Once again, it was determined using point \( s_x \) and point \( c \). In this case, point \( s_x \) and point \( c \) are not only rotated by \( \chi \), but also rotated by \( \theta_A \) and \( \theta_B \). Then, Eq. (14) and Eq. (15) changed to become,
Once $s_X$ and $c$ were determined, then the actual cutter tooth orientations ($\theta_{A_x}$, $\theta_{B_x}$) could be defined using Eq. (17) – Eq.(19). Finally the transformation operator to define the orientation and position of cutter tooth when the tool with helical angle with respect to $\theta_{A_x}$ and $\theta_{B_x}$ was defined as follow,

$$[M]_h = \text{Rot} \left( X, \theta_{A_x} \right) \times \text{Rot} \left( Y, \theta_{B_x} \right)$$

(22)

The swept Curve of Helical Cutting Tool

For non-solid cutting tool, the helical angle is not only change the orientation of cutter tooth, but also change the shape of swept surface. As illustrated in Figure 3a, the helical angle makes the radius of swept surface at the upper side is different to the one at the lower side.

The radius of swept surface equal to $R$ is only at the lowest side. Meanwhile at the upper side, the radius enlarges gradually as the increasing of the tool axial height ($l$). Therefore, the radius of swept surface as a function of axial height, $R_x(l)$, was defined by referring to Figure 3b as follows,

$$\psi = \tan^{-1} \left( \frac{(r - r \cos \chi) \sin \chi}{(r_m + r \sin \chi)} \right)$$

(23)

$$s_X(x_{s_x}, y_{s_x}, z_{s_x}) = \text{Rot} \left( X, \theta_A \right) \times \text{Rot} \left( Y, \theta_B \right) \times \text{Rot} \left( Z, \varphi \right) \times \text{Rot} \left( Y, \chi \right). [s] \quad (20)$$

$$c(x_c, y_c, z_c) = \text{Rot} \left( X, \theta_A \right) \times \text{Rot} \left( Y, \theta_B \right) \times \text{Rot} \left( Z, \varphi \right). [c] \quad (21)$$
$$R_x (\lambda) = \frac{r_m + r \sin \lambda}{\cos \psi}$$  \hspace{1cm} (24)

$$G_{R_x}(\varphi; \lambda) = \begin{bmatrix} R_x \sin \varphi \\ R_x \cos \varphi \\ (r - r \cos \lambda) \cos \chi \end{bmatrix}$$  \hspace{1cm} (25)

where $0 < \lambda < 90$ and $\psi$ is the lag angle as can be seen in Figure 2a. The lag angle is the rotation angle of $s_x$ relative to the rotation angle of $s$.

**PATH INTERSECTION POINT**

The scallop height ($h_s$) was defined as the normal length between the path intersection point (PIP) to the part surface normal ($p_n$). The PIP is an intersection point between the swept curve of current cutting path and that of adjacent cutting path. The equation to obtain the path intersection point were derived by referring to Figure 4. Since the tool orientation was set without tilt angle, then, the angle of cutter contact point ($\tau$), which is the deviation angle of cutting tool relative to the surface normal, was similar. It also made the PIP located in the middle of point $CC_1$ and $CC_2$. The distance of intersection point to the cutter contact point (CC-point), which is represented by $m$ and the angle of the CC point relative to the part surface was calculated by,

$$m = \frac{|CC_1 - CC_2|}{2}$$  \hspace{1cm} (26)

$$\tau = \sin^{-1}(m/R_1)$$  \hspace{1cm} (27)

where $R_1 = \sqrt{R_x^2 + R_y^2}$. $R_x$ and $R_y$ were the radius of part surface at instantaneous tool position that were calculated using the method as presented by Kiswanto et al. (2015). In this method, the part surface normal ($p_n$) was defined mathematically using a set of discrete
normal vectors. At any instantaneous tool position, the part surface was defined as a combination of a convex, concave, flat or sloped surface.

Regarding to the cutter orientation by the angle of CC-point (τ), then the coordinate of swept point in CCF was mapped to LCF. The mapping coordinate system was performed using the following equation,

\[
\begin{bmatrix}
  x_{Ic} \\
  y_{Ic} \\
  z_{Ic}
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos \tau & -\sin \tau \\
  0 & \sin \tau & \cos \tau
\end{bmatrix}
\times G_{tx} (\varphi_{Ic} ; \lambda_{Ic})
\]

(28)

The coordinate of \( I_c \) \( (x_{Ic}, y_{Ic}, z_{Ic}) \) could be determined after the toroidal angle of PIP (\( \lambda_{Ic} \)) was defined. With respect to CCF, \( y_{Ic} = m \). Since \( y_{Ic} \) was identified, \( \lambda_{Ic} \) was then defined by extracting Eq.(28) only for \( y_{Ic} \) as follow,

\[
y_{Ic} = (r_m + r \sin \lambda_{Ic}) \cos \psi_T \cos \varphi_{Ic} \cos \tau - (r - r \cos \lambda_{Ic}) \cos \chi \sin \tau
\]

(29)

There were three unknown variables exist in Eq.(29), \( \cos \psi_T, \lambda_{Ic} \) and \( \cos \varphi_{Ic} \). Therefore, \( \cos \psi_T \) and \( \cos \varphi_{Ic} \) need to be converted so that only \( \lambda_{Ic} \) is remaining unknown.

By rearranging Eq. (11), then \( \cos \varphi_{Ic} \) was expressed by,

\[
\cos \varphi_{Ic} = \frac{\sqrt{((V_T \cdot u) \sin \lambda_{Ic})^2 - ((V_T \cdot w) \cos \lambda_{Ic})^2}}{(V_T \cdot u) \sin \lambda_{Ic}}
\]

(30)

Meanwhile \( \cos \psi_T \) was obtained by rearranging Eq. (23) to become,
\[ \cos \psi_T = \left[ \frac{(r_m + r \sin \lambda_{lc})}{\sqrt{\left( r - r \cos \lambda_{lc} \sin \chi \right)^2 + \left( r_m + r \sin \lambda_{lc} \right)^2}} \right] \]  

(31)

After converting \( \cos(\varphi_{lc}) \) and \( \cos \psi_T \) in Eq. (29) by \( \cos(\varphi_{lc}) \) in Eq. (30) and \( \cos \psi_T \) in Eq. (31), finally, Eq. (29) yielded to become a polynomial equation as follow,

\[
\begin{align*}
(a^2)t^8 + (2ab)t^7 + (2ac + b^2 + f^2)t^6 + (2ad + 2bc)t^5 \\
+ (2ae + 2bd + c^2 - f^2 + 2fg)t^4 + (2be + 2cd)t^3 \\
+ (2ce + d^2 - 2fg + g^2)t^2 + (2de)t + (e^2 - g^2)
\end{align*}
\]

(32)

where,

\[ t = \sin \lambda_{lc} \]

\[ a = [(r^2 \cos^2 \tau)(1 - \sin^2 \chi) + r^2(V_T \cdot u)^2 \sin^2 \tau] \]

\[ b = [2r_m r \cos^2 \tau] \]

\[ c = \left[ \frac{(r^2 \cos^2 \tau \sin^2 \chi)(2 - (V_T \cdot w)^2) + (\cos^2 \tau)(r_m^2 - r^2(V_T \cdot w)^2)}{(V_T \cdot u)^2(-s^2 - 2r \cos \chi \sin \tau - 2r^2 \cos^2 \chi \sin^2 \tau)} \right] \]

(33)

\[ d = [-2r_m (V_T \cdot w)^2 \cos^2 \tau)] \]

\[ e = [(V_T \cdot w)^2 \cos^2 \tau) (-2r^2 \sin^2 \chi - r_m^2)] \]

\[ f = [(2r(V_T \cdot u)^2 \cos \chi \sin \tau)(-s - r \cos \chi \sin \tau) \\
+ (2r^2 \cos^2 \tau \sin^2 \chi)] \]

\[ g = [-2r^2(V_T \cdot w)^2 \cos^2 \tau \sin^2 \chi)] \]

There were eight possibilities of \( t \) could be generated by Eq. (32). Among them,
however, only one $t$ that appropriate to define $\lambda_{l_c}$ for defining the PIP. The appropriate one was chosen by using the following rules: a) $t$ must be within 0 and 1, b) $\lambda_{l_c}$ that gives $y_{l_c} = s$ will be selected. Once $\lambda_{l_c}$ was obtained, then the rotation angle could be determined using Eq. (11). The coordinate of PIP, $l_c \left(x_{l_c}, y_{l_c}, z_{l_c}\right)$, was defined by,

$$I_c \left(x_{l_c}, y_{l_c}, z_{l_c}\right) = \left[M\right]_h G_T \left(\varphi_{l_c}, \lambda_{l_c}\right)$$

(34)

Finally, the scallop height ($h$) was defined using the equation below,

$$h_s = \sqrt{I_{l_c}^2 - p_n^2}$$

(35)

IMPLEMENTATION AND DISCUSSION

All the equation derived in this study were used to develop a program simulation using MATLAB. The proposed method in this study was called Grazing Toroidal Approximation (GTA). In this section, the applicability of the proposed method to calculate the scallop height was checked. In the first test, the drawback of previous analytical studies in using ellipse to represent the swept curve was proven. The second test demonstrates the ability of GTA to calculate the scallop height. Finally, the accuracy of the proposed method was examined by comparing the scallop height obtained using Siemens-NX.

Grazing Toroidal Approximation vs Ellipse Curve Approximation

The proposed method was tested to check the applicability in generating the swept curve on the toroidal cutter as well as the influence of inclination angle to the shape of swept curve. In this test, a toroidal cutter with diameter 20 mm and a minor radius 5 mm was used as the cutting tool. The illustration of the swept curve on the toroidal cutter is displayed in Figure 5a.
Figure 5b shows a sample of the swept curve on cutter when the inclination angle exists, which was then projected into 2D. The projected swept curves for various inclination angle are depicted in Figure 5c. It can be seen that the shape of swept curve was much influenced by the inclination angle. When the inclination angle was set negative, the swept curve was located at the back of cutter tooth, as shown in Figure 5c. The toroidal angle of swept curve tends to increase when the inclination angle was increased. Figure 5b compares the shape of projected curve and ellipse curve for various inclination angle. As mentioned in Introduction that one of the drawback of the existing analytical method in scallop calculation is: mostly the researchers used parametric equation of ellipse to represent the swept curve. It can be seen that the shape of projected curve was very dynamic and it cannot be approximated by ellipse when the inclination was small. From a series of test, it was found that the projected curve coincides precisely with ellipse curve when the tool used large inclination angle (more than 40°), as shown in Figure 5d. In the real machining, however, large inclination angle is avoided. Hence, it proved that ellipse curve approximation method for toroidal cutter tend to produce error.

Implementation of the Grazing Toroidal Approximation

In this section, the applicability of the proposed method was verified for a model test as can be seen in Figure 6a. For simplifying the verification process, the test was performed using large step-over so that large scallop obtained. In this test, 15 mm step-over was selected. The machining condition set in the test were feedrate 0.3 mm/tooth and spindle speed 5000 rpm. A two teeth toroidal cutter with helical angle 10, diameter 20 mm and a minor radius of 5 mm was used as the cutting tool. Using the GTA, the shape of machined surface can be generated as can be seen in Figure-6b. From Figure 6a and Figure 6b can be seen that the shape of ma-
chined surface generated using Siemens-NX resembled the shape of machined surface generated using the program simulation. The shape of scallop can also be seen in more detail in Figure 6b. Moreover, the coordinate of the intersection point can be determined and hence the scallop height can be calculated. The scallop height that were generated using the GTA was presented in Figure 6c. The orientation of cutting tool and the shape of part surface were continuously changed during the machining process that causing the scallop height fluctuate during the machining process.

To ensure the ability of the method in calculating the scallop height, another test using a model as depicted in Figure 7a was performed. The milling condition set in this test were feedrate 0.3 mm/tooth, and spindle speed 7000 rpm. The tool used in the test was a two teeth toroidal cutter with diameter 25 mm and a minor radius of 5 mm. The inclination angle was set decrease gradually during ramp-up machining process, and then increase gradually during ramp-down milling. Using the same part model and cutting tool, the machining tests were performed using two different step-over, 20 mm and 10 mm. The shape of machined surface generating using Siemens-NX was shown in Figure 7a. Using the program simulation, the shape of machined surface can be generated, as shown in Figure 7b. The detail of scallop including the intersection point and the method to measure the scallop height was depicted in Figure 7c. The shape of cut produced by program simulation, as shown in Figure 7b, resembled the shape of machined surface generated using Siemens-NX, as presented in Figure 7a. The scallop height for every CC-points was calculated using the program simulation and the results for two step-over were presented in Figure 7d. From this figure can be concluded that increasing the step-over will increase the scallop height.

**Verification of the the Grazing Toroidal Approximation**

The proposed method was verified using the commercial software Siemens-NX. The shape of part surface after machining using Siemens-NX is presented in Figure-6a. Although
the shape of machined surfaces was similar, the accuracy of the developed method had to be verified. The verification was carried-out by comparing the scallop height generated using the proposed method with those measured using Siemens-NX. The method to measure cutter workpiece engagement in Siemens-NX was explained by Kiswanto et al. (2014a) and Kiswanto et al., (2015).

Based on the tool path data (G-Code), the geometry of swept volume can be constructed using Siemens-NX. Then, the machined surface was obtained by extracting the workpiece material using the swept volume model. After the machined surface was obtained, then the coordinate of a point on the machined surface can be checked. Finally, the scallop height could be determined by calculating the distance between an intersection point on the machined surface to the designed surface. The scallop height of every cc-points for one tool pass were generated, and the result are presented in Figure-6c. From this graph can be seen that the deviation of the verification data is relatively small. In general, the errors were less than 7%.

Test on The Effect of Helical Angle and Inclination Angle to Scallop Height

The ability of the proposed method to check the effect of helical angle to the scallop was also performed. For verification purpose, a part model as presented in Figure 8a was examined. The machining condition set in the test were feedrate 0.3 mm/tooth, step-over 10.49 mm and spindle speed 5000 rpm. Toroidal cutter with diameter 20 mm and a corner radius of 5 mm was used as the cutting tool. The inclination angle was set to decrease gradually during ramp-up machining process. In this test, the scallop height produced by the toroidal tool was tested using five different helical angles: 0, 10, 20, 30, 40.

The shape of machined surface for one tool pass is presented in Figure 8b. The magnitude of scallop height for all variables used in the test were presented in Figure 8c. From this fig-
ure can be seen that the scallop height decreased gradually as the inclination angle decreased during the ramp-up process. The same result was also demonstrated by previous test as shown in Figure 7d. The scallop height decrease gradually when the inclination angle decrease during ramp-up process (from CC-point 0 to CC-point 12). Then, the scallop increases gradually when the inclination angle increase during ramp-down process (from CC-point 13 to 24). The scallop height during ramp-down process show larger trend than that ramp-up process.

From Figure 8c can also be seen that helical angle gives significant effect to the scallop height. At the same machining condition, the helical angle tend to decrease the scallop height. This effect was occurred due to the change of swept surface caused by the existance of helical angle, as discussed in previous section (Fig. 3). The increasing of cutter diameter at the upper side made the intersection between the cutting tool in two subsequent tool pass produced smaller uncut material.

**CONCLUSION**

In this study, the Grazing Analytical Approximation method was extended by taken into consideration the effect of helical angle. This method was developed to generate the scallop height for a toroidal cutter during five-axis milling. Several important conclusions can be drawn as follow;

a. The test proved that the extended GTA is applicable to define the scallop height of helical toroidal cutter in five-axis milling process.

b. The verification test, which was performed by comparing the scallop height obtained using the GTA with those measured using Siemens-NX, proved that the GTA is accurate.
c. The test was also shown that increasing helical angle will decrease the scallop height. On the other hand, decreasing the inclination angle will decrease the scallop height.

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REFERENCES


Figure 1  a) Geometry of cutting tool, b) tool orientation relative to GCF, c) three coordinate systems

Figure 2 The orientation of cutter tooth with respect to rotation angle due to the effect of helical angle, b) the orientation of cutter tooth,
Figure 3 a) the shape of swept surface using helical cutting tooth, b) lag angle

Figure 4 Intersection point of adjacent tool path, a) front view, b) side view
Figure 5  a) Swept curve on toroidal cutter, b) projected swept curve, c) the shape of projected swept curve for various inclination angle, d) comparison between projected swept curve and ellipse curve.

Figure 6  a) Model test, b) machined surface generated using proposed method, b) machined surface generated using proposed method, c) calculated and measured scallop height.
Figure 7 a) Model test generated using Siemens-NX, b) machined surface generated using proposed method, c) the detail of scallop, d) the scallop height for two step-over

Figure 8 a) model test, b) shape of scallop, c) scallop height for various helical angles